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SOME PROPERTIES OF NEUTROSOPHIC CUBIC SOFT SETS

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Abstract:

The notion of Neutrosophic cubic soft sets P-union, P-intersection, are introduced and their related properties are investigated. We discussed about the internal and external neutrosophic cubic soft sets.

Key Words: Neutrosophic Cubic Soft Set, P-Union and P-Intersection & Internal and External Neutrosophic Cubic Soft Sets.

1. Introduction:

Neutrosophic set theory [5] extended the concept of cubic sets to the neutrosophic sets. They introduced the notions of truth-internal (indeterminacy-internal, falsity-internal) neutrosophic cubic sets and truth-external (indeterminacy-external, falsity-external) neutrosophic cubic sets and investigate related properties. In this paper , we introduced the definition of Neutrosophic cubic soft sets(NCSS)and proved some properties, We give a condition for the P-intersection of truth -external (resp. indeterminacy-external and falsity-external) neutrosophic cubic soft sets to be truth-external NCSS. We give an example to show that the P-union of falsity-external neutrosophic cubic soft sets may not be a NCSS. We provide a condition for the P-union of falsity-external (resp.indeterminacy-external and truth-external) neutrosophic cubic soft sets to be a falsity-external (resp. indeterminacy-external and truth-external) NCSS. Also we state the condition for the P-intersection of two neutrosophic cubic soft sets to be both a falsity-internal NCSS and a falsity external NCSS. We provide a condition for the P-union of two truth-external neutrosophic cubic soft sets to be a truth-internal NCSS.

2. Preliminaries:

Definition: 2.1

A pair $(\tilde{F}, I)$ is called cubic soft set over X if and only if $\tilde{F}$ is a mapping of I$(\subseteq E)$ into the set of all cubic sets in X, i.e., $\tilde{F}: I \rightarrow CP(X)$ where I is any subset of parameter's set E, X is an initial universe set and $CP(X)$ is the collection of all cubic sets in X. Here we denote and define cubic soft set as $(\tilde{F}, I) = \tilde{F}(e_i) = \mathcal{A} = \{< x, A_{e_i}(x), \lambda_{e_i}(x) >; x \in X\} e_i \in I$ in this set corresponding to each $e_i \in I$, $\mathcal{A} = \{< x, A_{e_i}(x), \lambda_{e_i}(x) >; x \in X\}$ is a cubic set in X in which $A_{e_i}(x)$ is an interval valued fuzzy set (briefly, an IVF set) and $\lambda_{e_i}(x)$ is a fuzzy set.

Definition: 2.2

Let X be a non-empty set. An interval neutrosophic set (INS) in X is a structure of the form:

$$\mathcal{A} = \{< x, A_T(x), A_I(x), A_F(x) >; x \in X\}$$

Where A_T, A_I and A_F are interval-valued fuzzy sets in X, which are called an interval truth membership function, an interval indeterminacy membership function and an interval falsity membership function, respectively.

Let X be a non-empty set. A neutrosophic cubic set (NCS) in X is a pair Å=($\mathcal{A}$,Λ) where $= \{< x, A_T(x), A_I(x), A_F(x) >; x \in X\}$ is an interval neutrosophic set in X and

$\Lambda = \{< x, \lambda_T(x), \lambda_I(x), \lambda_F(x) >; x \in X\}$ is a neutrosophic set in X.

Definition: 2.3

Let X be a non-empty set. A neutrosophic cubic set Å=($\mathcal{A}$, Λ) is said to be,

- ✓ truth-internal(briefly, T-internal) if the following inequality is valid $(x \in X)(A_T^-(x) \leq \lambda_T(x) \leq A_T^+(x))$,
- ✓ indeterminacy-internal(briefly, I-internal) if the following inequality is valid $(x \in X)(A_I^-(x) \leq \lambda_I(x) \leq A_I^+(x))$,
- ✓ falsity-internal(briefly, F-internal) if the following inequality is valid $(x \in X)(A_F^-(x) \leq \lambda_F(x) \leq A_F^+(x))$,

Definition: 2.4

Let X be a non-empty set. A neutrosophic cubic set Å=($\mathcal{A}$, Λ) is said to be,

- ✓ truth-external(briefly, T-external) if the following inequality is valid $(x \in X)(\lambda_T(x) \notin (A_T^-(x), A_T^+(x)))$.
- ✓ indeterminacy-external(briefly, I-external) if the following inequality is valid

$(x \in X)(\lambda_I(x) \notin (A_I^-(x), A_I^+(x)))$.

- ✓ falsity-external(briefly, F-external) if the following inequality is valid $(x \in X)(\lambda_F(x) \notin (A_F^-(x), A_F^+(x)))$.

Definition: 2.5

Let X be a non-empty set. A neutrosophic cubic soft set (NCSS) in X is a pair $(\tilde{F}, I)$ =Å = ($\mathcal{A}$, Λ) where, $\mathcal{A} = \{< x, A_{T_{e_i}}(x), A_{I_{e_i}}(x), A_{F_{e_i}}(x) >; x \in X\}$ is an interval neutrosophic soft set in X and $\Lambda = \{< x, \lambda_{T_{e_i}}(x), \lambda_{I_{e_i}}(x), \lambda_{F_{e_i}}(x) >; x \in X\}$ is a neutrosophic soft set.

Theorem: 2.6

Let $(\tilde{F}, I)$=Å and $(\tilde{G}, J)$=Ḃ be T-external neutrosophic cubic soft sets in X such that,

$$\max\{\min\{A^+_{T_{e_i}}(x), B^-_{T_{e_i}}(x)\}, \min\{A^-_{T_{e_i}}(x), B^+_{T_{e_i}}(x)\}\} < (\lambda_{T_{e_i}} \wedge \psi_{T_{e_i}})(x)$$
$$\leq \min\{\max\{A^+_{T_{e_i}}(x), B^-_{T_{e_i}}(x)\}, \max\{A^-_{T_{e_i}}(x), B^+_{T_{e_i}}(x)\}\} \qquad (1)$$

for all $x \in X$ .Then $(\tilde{F}, I) \cap_P (\tilde{G}, J)$ is a T-external neutrosophic cubic soft set in X.

Proof:

Consider, $(\tilde{F}, I) \cap_P (\tilde{G}, J) = (\tilde{H}, C)$ where $C = I \cap J$

$$\tilde{H}(e_i) = \tilde{F}(e_i) \qquad if \quad e_i \in I - J$$
$$= \tilde{G}(e_i) \qquad if \quad e_i \in J - I$$
$$= \tilde{F}(e_i) \wedge_P \tilde{G}(e_i) \quad if \quad e_i \in I \cap J.$$

Where $\tilde{F}(e_i) \wedge_P \tilde{G}(e_i)$ is defined as,

$$\tilde{F}(e_i) \wedge_P \tilde{G}(e_i) = \{< x, r\min\{A_{T_{e_i}}(x), B_{T_{e_i}}(x)\}, (\lambda_{T_{e_i}}(x) \wedge \psi_{T_{e_i}}(x)) >: x \in X\}, e_i \in I \cap J.$$

Take,
$$\alpha_{e_i} = \min\{\max\{A^+_{T_{e_i}}(x), B^-_{T_{e_i}}(x)\}, \max\{A^-_{T_{e_i}}(x), B^+_{T_{e_i}}(x)\}\}$$
$$\beta_{e_i} = \max\{\min\{A^+_{T_{e_i}}(x), B^-_{T_{e_i}}(x)\}, \min\{A^-_{T_{e_i}}(x), B^+_{T_{e_i}}(x)\}\}$$

Then
$$\alpha_{e_i} = A^-_{T_{e_i}}(x),\ \alpha_{e_i} = B^-_{T_{e_i}}(x), \alpha_{e_i} = A^+_{T_{e_i}}(x)\, or\, \alpha_{e_i} = B^+_{T_{e_i}}(x)$$

It is possible to consider, the cases $\alpha_{e_i} = A^-_{T_{e_i}}(x)$ and $\alpha_{e_i} = A^+_{T_{e_i}}(x)$ only because the remaining cases are similar to these cases, if $\alpha_{e_i} = A^-_{T_{e_i}}(x)$,then, $B^-_{T_{e_i}}(x) \leq B^+_{T_{e_i}}(x) \leq A^-_{T_{e_i}}(x) \leq A^+_{T_{e_i}}(x)$

Thus, $\beta_{e_i} = B^+_{T_{e_i}}(x)$, and so $B^-_{T_{e_i}} = (A_{T_{e_i}} \cap B_{T_{e_i}})^-(x) \leq (A_{T_{e_i}} \cap B_{T_{e_i}})^+(x) = B^+_{T_{e_i}}(x) = \beta_{e_i} < (\lambda_{T_{e_i}} \wedge \psi_{T_{e_i}})(x)$

Hence, $(\lambda_{T_{e_i}} \wedge \psi_{T_{e_i}})(x) \notin ((A_{T_{e_i}} \cap B_{T_{e_i}})^-(x), (A_{T_{e_i}} \cap B_{T_{e_i}})^+(x))$

If $\alpha_{e_i} = A^+_{T_{e_i}}(x)$, then, $B^-_{T_{e_i}}(x) \leq A^+_{T_{e_i}}(x) \leq B^+_{T_{e_i}}(x)$ and thus $\beta_{e_i} = \max\{A^-_{T_{e_i}}(x), B^-_{T_{e_i}}(x)\}$ Suppose that, $\beta_{e_i} = A^-_{T_{e_i}}(x)$, then

$$B^-_{T_{e_i}}(x) \leq A^-_{T_{e_i}}(x) < (\lambda_{T_{e_i}} \wedge \psi_{T_{e_i}})(x) \leq A^+_{T_{e_i}}(x) \leq B^+_{T_{e_i}}(x) \qquad (2)$$

It follows that,
$$B^-_{T_{e_i}}(x) \leq A^-_{T_{e_i}}(x) < (\lambda_{T_{e_i}} \wedge \psi_{T_{e_i}})(x) < A^+_{T_{e_i}}(x) \leq B^+_{T_{e_i}}(x) \qquad (3)$$

Or
$$B^-_{T_{e_i}}(x) \leq A^-_{T_{e_i}}(x) < (\lambda_{T_{e_i}} \wedge \psi_{T_{e_i}})(x) = A^+_{T_{e_i}}(x) \leq B^+_{T_{e_i}}(x) \qquad (4)$$

The case (3) induces a contradiction. The case (4) implies that,

$$(\lambda_{T_{e_i}} \wedge \psi_{T_{e_i}})(x) \notin ((A_{T_{e_i}} \cap B_{T_{e_i}})^-(x), (A_{T_{e_i}} \cap B_{T_{e_i}})^+(x))$$

Since
$$(\lambda_{T_{e_i}} \wedge \psi_{T_{e_i}})(x) = A^+_{T_{e_i}}(x) = (A_{T_{e_i}} \cap B_{T_{e_i}})^+(x)$$

Now, if $\beta_{e_i} = B^-_{T_{e_i}}(x)$, then $A^-_{T_{e_i}}(x) \leq B^-_{T_{e_i}}(x) < (\lambda_{T_{e_i}} \wedge \psi_{T_{e_i}})(x) \leq A^+_{T_{e_i}}(x) \leq B^+_{T_{e_i}}(x)$ (5)

Hence we have,
$$A^-_{T_{e_i}}(x) \leq B^-_{T_{e_i}}(x) < (\lambda_{T_{e_i}} \wedge \psi_{T_{e_i}})(x) < A^+_{T_{e_i}}(x) \leq B^+_{T_{e_i}}(x) \qquad (6)$$

Or
$$A^-_{T_{e_i}}(x) \leq B^-_{T_{e_i}}(x) < (\lambda_{T_{e_i}} \wedge \psi_{T_{e_i}})(x) = A^+_{T_{e_i}}(x) \leq B^+_{T_{e_i}}(x) \qquad (7)$$

The case (6) induces a contradiction. Then case (7) induces,

$$(\lambda_{T_{e_i}} \wedge \psi_{T_{e_i}})(x) \notin ((A_{T_{e_i}} \cap B_{T_{e_i}})^-(x), (A_{T_{e_i}} \cap B_{T_{e_i}})^+(x))$$

Consequently, we note that $(\tilde{F}, I) \cap_P (\tilde{G}, J)$ is an T-external neutrosophic cubic soft set in X.

Theorem: 2.7

Let $(\tilde{F}, I)$=Å and $(\tilde{G}, J)$=Ḃ be F-external neutrosophic cubic soft sets in X such that,

$$\max\{\min\{A^+_{F_{e_i}}(x), B^-_{F_{e_i}}(x)\}, \min\{A^-_{F_{e_i}}(x), B^+_{F_{e_i}}(x)\}\} < (\lambda_{F_{e_i}} \wedge \psi_{F_{e_i}})(x)$$
$$\leq \min\{\max\{A^+_{F_{e_i}}(x), B^-_{F_{e_i}}(x)\}, \max\{A^-_{F_{e_i}}(x), B^+_{F_{e_i}}(x)\}\} \qquad (1)$$

for all $x \in X$ .Then $(\tilde{F},I)\cap_P(\tilde{G},J)$ is a F-external neutrosophic cubic soft set in X.

Proof:

Consider, $(\tilde{F},I)\cap_P(\tilde{G},J)=(\tilde{H},C)$ where $C=I\cap J$

$$\begin{aligned}\tilde{H}(e_i)&=\tilde{F}(e_i) && if \quad e_i\in I-J\\ &=\tilde{G}(e_i) && if \quad e_i\in J-I\\ &=\tilde{F}(e_i)\wedge_P\tilde{G}(e_i) && if \quad e_i\in I\cap J.\end{aligned}$$

Where $\tilde{F}(e_i)\wedge_P\tilde{G}(e_i)$ is defined as,

$$\tilde{F}(e_i)\wedge_P\tilde{G}(e_i)=\left\{<x,r\min\left\{A_{Fe_i}(x),B_{Fe_i}(x)\right\},(\lambda_{Fe_i}(x)\wedge\psi_{Fe_i}(x))>:x\in X\right\},e_i\in I\cap J.$$

Take, $\alpha_{e_i}=\min\left\{\max\left\{A^+_{F_{e_i}}(x),B^-_{F_{e_i}}(x)\right\},\max\left\{A^-_{F_{e_i}}(x),B^+_{F_{e_i}}(x)\right\}\right\}$

$\beta_{e_i}=\max\left\{\min\left\{A^+_{F_{e_i}}(x),B^-_{F_{e_i}}(x)\right\},\min\left\{A^-_{F_{e_i}}(x),B^+_{F_{e_i}}(x)\right\}\right\}$

Then $\alpha_{e_i}=A^-_{F_{e_i}}(x),\ \alpha_{e_i}=B^-_{F_{e_i}}(x),\alpha_{e_i}=A^+_{F_{e_i}}(x)\,or\,\alpha_{e_i}=B^+_{F_{e_i}}(x)$

It is possible to consider, the cases $\alpha_{e_i}=A^-_{F_{e_i}}(x)$ and $\alpha_{e_i}=A^+_{F_{e_i}}(x)$ only because the remaining cases are similar to these cases, if $\alpha_{e_i}=A^-_{F_{e_i}}(x)$,then $B^-_{F_{e_i}}(x)\le B^+_{F_{e_i}}(x)\le A^-_{F_{e_i}}(x)\le A^+_{F_{e_i}}(x)$

Thus, $\beta_{e_i}=B^+_{F_{e_i}}(x)$, and so, $B^-_{F_{e_i}}=(A_{F_{e_i}}\cap B_{F_{e_i}})^-(x)\le(A_{F_{e_i}}\cap B_{F_{e_i}})^+(x)=B^+_{F_{e_i}}(x)=\beta_{e_i}<(\lambda_{F_{e_i}}\wedge\psi_{F_{e_i}})(x)$

Hence, $(\lambda_{F_{e_i}}\wedge\psi_{F_{e_i}})(x)\notin((A_{F_{e_i}}\cap B_{F_{e_i}})^-(x),(A_{F_{e_i}}\cap B_{F_{e_i}})^+(x))$

If $\alpha_{e_i}=A^+_{F_{e_i}}(x)$, then, $B^-_{F_{e_i}}(x)\le A^+_{F_{e_i}}(x)\le B^+_{F_{e_i}}(x)$ and thus,

$\beta_{e_i}=\max\left\{A^-_{F_{e_i}}(x),B^-_{F_{e_i}}(x)\right\}$ Suppose that, $\beta_{e_i}=A^-_{F_{e_i}}(x)$ then,

$$B^-_{F_{e_i}}(x)\le A^-_{F_{e_i}}(x)<(\lambda_{F_{e_i}}\wedge\psi_{F_{e_i}})(x)\le A^+_{F_{e_i}}(x)\le B^+_{F_{e_i}}(x) \qquad (2)$$

It follows that,
$$B^-_{F_{e_i}}(x)\le A^-_{F_{e_i}}(x)<(\lambda_{F_{e_i}}\wedge\psi_{F_{e_i}})(x)< A^+_{F_{e_i}}(x)\le B^+_{F_{e_i}}(x) \qquad (3)$$

Or
$$B^-_{F_{e_i}}(x)\le A^-_{F_{e_i}}(x)<(\lambda_{F_{e_i}}\wedge\psi_{F_{e_i}})(x)= A^+_{F_{e_i}}(x)\le B^+_{F_{e_i}}(x) \qquad (4)$$

The case (3) induces a contradiction. The case (4) implies that,

$$(\lambda_{F_{e_i}}\wedge\psi_{F_{e_i}})(x)\notin((A_{F_{e_i}}\cap B_{F_{e_i}})^-(x),(A_{F_{e_i}}\cap B_{F_{e_i}})^+(x))$$

Since $(\lambda_{F_{e_i}}\wedge\psi_{F_{e_i}})(x)=A^+_{F_{e_i}}(x)=(A_{F_{e_i}}\cap B_{F_{e_i}})^+(x)$

Now, if $\beta_{e_i}=B^-_{F_{e_i}}(x)$ then $A^-_{F_{e_i}}(x)\le B^-_{F_{e_i}}(x)<(\lambda_{F_{e_i}}\wedge\psi_{F_{e_i}})(x)\le A^+_{F_{e_i}}(x)\le B^+_{F_{e_i}}(x) \qquad (5)$

Hence we have,
$$A^-_{F_{e_i}}(x)\le B^-_{F_{e_i}}(x)<(\lambda_{F_{e_i}}\wedge\psi_{F_{e_i}})(x)< A^+_{F_{e_i}}(x)\le B^+_{F_{e_i}}(x) \qquad (6)$$

Or
$$A^-_{F_{e_i}}(x)\le B^-_{F_{e_i}}(x)<(\lambda_{F_{e_i}}\wedge\psi_{F_{e_i}})(x)= A^+_{F_{e_i}}(x)\le B^+_{F_{e_i}}(x) \qquad (7)$$

The case (6) induces a contradiction. Then case (7) induces,

$$(\lambda_{F_{e_i}}\wedge\psi_{F_{e_i}})(x)\notin((A_{F_{e_i}}\cap B_{F_{e_i}})^-(x),(A_{F_{e_i}}\cap B_{F_{e_i}})^+(x))$$

Consequently, we note that $(\tilde{F},I)\cap_P(\tilde{G},J)$ is an F-external neutrosophic cubic soft set in X.

Similarly we have the following theorem.

Theorem: 2.8

Let $(\tilde{F},I)$=Ấ and $(\tilde{G},J)$=Ḃ be I-external neutrosophic cubic soft sets in X such that,

$$\max\left\{\min\left\{A^+_{I_{e_i}}(x),B^-_{I_{e_i}}(x)\right\},\min\left\{A^-_{I_{e_i}}(x),B^+_{I_{e_i}}(x)\right\}\right\}<(\lambda_{I_{e_i}}\wedge\psi_{I_{e_i}})(x)$$
$$\le\min\left\{\max\left\{A^+_{I_{e_i}}(x),B^-_{I_{e_i}}(x)\right\},\max\left\{A^-_{I_{e_i}}(x),B^+_{I_{e_i}}(x)\right\}\right\}$$

for all $x\in X$ .Then $(\tilde{F},I)\cap_P(\tilde{G},J)$ is a I-external neutrosophic cubic soft set in X.

Example: 2.9

Let $(\tilde{F},I)$=Ấ and $(\tilde{G},J)$=Ḃ be I-external neutrosophic cubic soft sets in X =[0,1] with the tabular representations in Tables 1 and 2, respectively,

Table 1: Tabular representation of (𝒜, Λ) =Ấ

X	$\mathcal{A}_{e1}$(x)	Λ_{e1}(x)
$0\le x\le 0.5$	([0.25,0.26],[0.2,0.3],[0.15,0.25])	(0.25,0.15,0.5x+0.5)
$0.5\le x\le 1$	([0.5,0.7],[0.5,0.6],[0.6,0.7])	(0.55,0.75,0.30)

$0 \le x \le 0.3$ $0.3 \le x \le 1$	([0.1,0.15],[0.2,0.25],[0.15,0.25]) ([0.1,0.2],[0.7,0.8],[0.6,0.7])	(0.25,0.15,0.1x+0.1) (0.55,0.70,0.30)

X	$\mathcal{A}_{e2}(x)$	$\Lambda_{e2}(x)$
$0 \le x \le 0.7$ $0.7 \le x \le 1$	([0.25,0.26],[0.2,0.5],[0.15,0.35]) ([0.7,0.8],[0.7,0.75],[0.8,0.9])	(0.25,0.45,0.7) (0.8,0.85,0.92)
$0 \le x \le 0.5$ $0.5 \le x \le 1$	([0.25,0.26],[0.2,0.3],[0.15,0.25]) ([0.5,0.7],[0.5,0.6],[0.6,0.7])	(0.25,0.15,0.5x+0.5) (0.55,0.70,0.30)

Table 2: Tabular representation of $(\mathfrak{B}, \psi) = \dot{B}$

X	$\mathfrak{B}_{e1}(x)$	$\psi_{e1}(x)$
$0 \le x \le 0.5$ $0.5 \le x \le 1$	([0.25,0.26],[0.2,0.3],[0.8,0.9]) ([0.5,0.7],[0.5,0.6],[0.1,0.2])	(0.25,0.15,0.40) (0.55,0.75, x)
$0 \le x \le 0.3$ $0.3 \le x \le 1$	([0.1,0.15],[0.2,0.25],[0.15,0.28]) ([0.1,0.2],[0.7,0.8],[0.7,0.8])	(0.25,0.15,0.1x+0.1) (0.55,0.70,0.40)

X	$\mathfrak{B}_{e2}(x)$	$\psi_{e2}(x)$
$0 \le x \le 0.7$ $0.7 \le x \le 1$	([0.25,0.26],[0.2,0.5],[0.25,0.35]) ([0.7,0.8],[0.7,0.75],[0.8,0.92])	(0.25,0.45,0.7) (0.8,0.85,0.92)
$0 \le x \le 0.5$ $0.5 \le x \le 1$	([0.25,0.26],[0.2,0.3],[0.18,0.30]) ([0.5,0.7],[0.5,0.6],[0.7,0.8])	(0.25,0.15,0.5x+0.5) (0.55,0.75,0.85)

X	$(\mathcal{A}_{e1} \cup \mathfrak{B}_{e1})(x)$	$(\Lambda_{e1} \cup \psi_{e1})(x)$
$0 \le x \le 0.5$ $0.5 \le x \le 1$	([0.25,0.26],[0.2,0.3],[0.8,0.9]) ([0.5,0.7],[0.5,0.6],[0.6,0.7])	(0.25,0.15,0.5x+0.5) (0.55,0.75, x)
$0 \le x \le 0.3$ $0.3 \le x \le 1$	([0.1,0.15],[0.2,0.25],[0.15,0.28]) ([0.1,0.2],[0.7,0.8],[0.7,0.8])	(0.25,0.15,0.1x+0.1) (0.55,0.70,0.40)

X	$(\mathcal{A}_{e2} \cup \mathfrak{B}_{e2})(x)$	$(\Lambda_{e2} \cup \psi_{e2})(x)$
$0 \le x \le 0.7$ $0.7 \le x \le 1$	([0.25,0.26],[0.2,0.5],[0.25,0.35]) ([0.7,0.8],[0.7,0.75],[0.8,0.92])	(0.25,0.45,0.7) (0.8,0.85,0.92)
$0 \le x \le 0.5$ $0.5 \le x \le 1$	([0.25,0.26],[0.2,0.3],[0.18,0.30]) ([0.5,0.7],[0.5,0.6],[0.7,0.8])	(0.25,0.15,0.5x+0.5) (0.55,0.75,0.85)

Then, $(\lambda_{F_{e_1}} \vee \psi_{F_{e_1}}) = 0.67 \in (0.67, 0.7) = (A_{F_{e_1}} \cup B_{F_{e_1}})^-(0.67), (A_{F_{e_1}} \cup B_{F_{e_1}})^+(0.67)$

$(\lambda_{F_{e_1}} \vee \psi_{F_{e_1}}) = 0.78 \in (0.7, 0.8) = (A_{F_{e_1}} \cup B_{F_{e_1}})^-(0.78), (A_{F_{e_1}} \cup B_{F_{e_1}})^+(0.78)$

$(\lambda_{F_{e_2}} \vee \psi_{F_{e_2}}) = 0.82 \in (0.8, 0.9) = (A_{F_{e_2}} \cup B_{F_{e_2}})^-(0.82), (A_{F_{e_2}} \cup B_{F_{e_2}})^+(0.82)$

$(\lambda_{F_{e_2}} \vee \psi_{F_{e_2}}) = 0.78 \in (0.7, 0.8) = (A_{F_{e_2}} \cup B_{F_{e_2}})^-(0.78), (A_{F_{e_2}} \cup B_{F_{e_2}})^+(0.78)$

and so the P-union $(\tilde{F}, I) \cup_P (\tilde{G}, J) = (\mathcal{A} \cup \mathfrak{B}, \Lambda \cup \psi)$ is not an F-external neutrosophic cubic soft set in X=[0,1].

Theorem: 2.10

Let $(\tilde{F}, I) = \acute{\mathring{A}}$ and $(\tilde{G}, J) = \dot{B}$ be F-external neutrosophic cubic soft sets in X such that,

$$\max\left\{\min\left\{A^+_{F_{e_i}}(x), B^-_{F_{e_i}}(x)\right\}, \min\left\{A^-_{F_{e_i}}(x), B^+_{F_{e_i}}(x)\right\}\right\} \le (\lambda_{F_{e_i}} \wedge \psi_{F_{e_i}})(x)$$
$$< \min\left\{\max\left\{A^+_{F_{e_i}}(x), B^-_{F_{e_i}}(x)\right\}, \max\left\{A^-_{F_{e_i}}(x), B^+_{F_{e_i}}(x)\right\}\right\} \qquad (1)$$

for all $x \in X$. Then $(\tilde{F}, I) \cup_P (\tilde{G}, J)$ is a F-external neutrosophic cubic soft set in X.

Proof:

Consider, $(\tilde{F}, I) \cup_P (\tilde{G}, J) = (\tilde{H}, C)$ where $C = I \cup J$

$$\begin{aligned}\tilde{H}(e_i) &= \tilde{F}(e_i) \quad &if \quad e_i \in I - J\\ &= \tilde{G}(e_i) \quad &if \quad e_i \in J - I\\ &= \tilde{F}(e_i) \vee_P \tilde{G}(e_i) \quad &if \quad e_i \in I \cap J.\end{aligned}$$

Where $\tilde{F}(e_i) \vee_P \tilde{G}(e_i)$ is defined as,

$$\tilde{F}(e_i) \vee_P \tilde{G}(e_i) = \{< x, r\max\{A_{F e_i}(x), B_{F e_i}(x)\}, (\lambda_{F e_i}(x) \vee \psi_{F e_i}(x)) >: x \in X\}, e_i \in I \cap J.$$

Take, $\alpha_{e_i} = \min\{\max\{A^+_{F_{e_i}}(x), B^-_{F_{e_i}}(x)\}, \max\{A^-_{F_{e_i}}(x), B^+_{F_{e_i}}(x)\}\}$

$$\beta_{e_i} = \max\{\min\{A^+_{F_{e_i}}(x), B^-_{F_{e_i}}(x)\}, \min\{A^-_{F_{e_i}}(x), B^+_{F_{e_i}}(x)\}\}$$

Then $\alpha_{e_i} = A^-_{F_{e_i}}(x), \alpha_{e_i} = B^-_{F_{e_i}}(x), \alpha_{e_i} = A^+_{F_{e_i}}(x) \, or \, \alpha_{e_i} = B^+_{F_{e_i}}(x)$

It is possible to consider, the cases $\alpha_{e_i} = A^-_{F_{e_i}}(x)$ and $\alpha_{e_i} = A^+_{F_{e_i}}(x)$ only because the remaining cases are similar to these cases, if $\alpha_{e_i} = A^-_{F_{e_i}}(x)$, then, $B^-_{F_{e_i}}(x) \le B^+_{F_{e_i}}(x) \le A^-_{F_{e_i}}(x) \le A^+_{F_{e_i}}(x)$

Thus, $\beta_{e_i} = B^+_{F_{e_i}}(x)$, and so $B^-_{F_{e_i}} = (A_{F_{e_i}} \cup B_{F_{e_i}})^-(x) = A^-_{F_{e_i}}(x) = \alpha_{e_i} > (\lambda_{F_{e_i}} \wedge \psi_{F_{e_i}})(x)$

Hence, $(\lambda_{F_{e_i}} \vee \psi_{F_{e_i}})(x) \notin ((A_{F_{e_i}} \cup B_{F_{e_i}})^-(x), (A_{F_{e_i}} \cup B_{F_{e_i}})^+(x))$

If $\alpha_{e_i} = A^+_{F_{e_i}}(x)$, then, $B^-_{F_{e_i}}(x) \le A^+_{F_{e_i}}(x) \le B^+_{F_{e_i}}(x)$ and thus

$\beta_{e_i} = \max\{A^-_{F_{e_i}}(x), B^-_{F_{e_i}}(x)\}$ Suppose that, $\beta_{e_i} = A^-_{F_{e_i}}(x)$ then

$$B^-_{F_{e_i}}(x) \le A^-_{F_{e_i}}(x) < (\lambda_{F_{e_i}} \vee \psi_{F_{e_i}})(x) \le A^+_{F_{e_i}}(x) \le B^+_{F_{e_i}}(x) \qquad (2)$$

It follows that,
$$B^-_{F_{e_i}}(x) \le A^-_{F_{e_i}}(x) < (\lambda_{F_{e_i}} \vee \psi_{F_{e_i}})(x) < A^+_{F_{e_i}}(x) \le B^+_{F_{e_i}}(x) \qquad (3)$$

Or
$$B^-_{F_{e_i}}(x) \le A^-_{F_{e_i}}(x) = (\lambda_{F_{e_i}} \vee \psi_{F_{e_i}})(x) < A^+_{F_{e_i}}(x) \le B^+_{F_{e_i}}(x) \qquad (4)$$

The case (3) induces a contradiction. The case (4) implies that, $(\lambda_{F_{e_i}} \vee \psi_{F_{e_i}})(x) \notin ((A_{F_{e_i}} \cup B_{F_{e_i}})^-(x), (A_{F_{e_i}} \cup B_{F_{e_i}})^+(x))$

Since $(\lambda_{F_{e_i}} \vee \psi_{F_{e_i}})(x) = A^-_{F_{e_i}}(x) = (A_{F_{e_i}} \cup B_{F_{e_i}})^-(x)$

Now, if $\beta_{e_i} = B^-_{F_{e_i}}(x)$ then, $A^-_{F_{e_i}}(x) \le B^-_{F_{e_i}}(x) \le (\lambda_{F_{e_i}} \vee \psi_{F_{e_i}})(x) \le A^+_{F_{e_i}}(x) \le B^+_{F_{e_i}}(x) \qquad (5)$

Hence we have,
$$A^-_{F_{e_i}}(x) \le B^-_{F_{e_i}}(x) < (\lambda_{F_{e_i}} \vee \psi_{F_{e_i}})(x) \le A^+_{F_{e_i}}(x) \le B^+_{F_{e_i}}(x) \qquad (6)$$

Or
$$A^-_{F_{e_i}}(x) \le B^-_{F_{e_i}}(x) = (\lambda_{F_{e_i}} \vee \psi_{F_{e_i}})(x) \le A^+_{F_{e_i}}(x) \le B^+_{F_{e_i}}(x) \qquad (7)$$

The case (6) induces a contradiction. Then case (7) induces,

$$(\lambda_{F_{e_i}} \vee \psi_{F_{e_i}})(x) \notin ((A_{F_{e_i}} \cup B_{F_{e_i}})^-(x), (A_{F_{e_i}} \cup B_{F_{e_i}})^+(x))$$

Consequently, we note that $(\tilde{F}, I) \cup_P (\tilde{G}, J)$ is an F-external neutrosophic cubic soft set in X.
Similarly we have the following theorems.

Theorem: 2.11

Let $(\tilde{F}, I)$=Á and $(\tilde{G}, J)$=Ḃ be T-external neutrosophic cubic soft sets in X such that,

$$\max\{\min\{A^+_{T_{e_i}}(x), B^-_{T_{e_i}}(x)\}, \min\{A^-_{T_{e_i}}(x), B^+_{T_{e_i}}(x)\}\} \le (\lambda_{T_{e_i}} \vee \psi_{T_{e_i}})(x)$$
$$< \min\{\max\{A^+_{T_{e_i}}(x), B^-_{T_{e_i}}(x)\}, \max\{A^-_{T_{e_i}}(x), B^+_{T_{e_i}}(x)\}\}$$

For all $x \in X$ .Then $(\tilde{F}, I) \cup_P (\tilde{G}, J)$ is a T-external neutrosophic cubic soft set in X.

Theorem: 2.12

Let $(\tilde{F}, I)$=Á and $(\tilde{G}, J)$=Ḃ be I-external neutrosophic cubic soft sets in X such that,

$$\max\{\min\{A^+_{I_{e_i}}(x), B^-_{I_{e_i}}(x)\}, \min\{A^-_{I_{e_i}}(x), B^+_{I_{e_i}}(x)\}\} \le (\lambda_{I_{e_i}} \vee \psi_{I_{e_i}})(x)$$
$$< \min\{\max\{A^+_{I_{e_i}}(x), B^-_{I_{e_i}}(x)\}, \max\{A^-_{I_{e_i}}(x), B^+_{I_{e_i}}(x)\}\}$$

For all $x \in X$ .Then $(\tilde{F}, I) \cup_P (\tilde{G}, J)$ is an I-external neutrosophic cubic soft set in X.

Theorem: 2.13

Let $(\tilde{F}, I)$=Á and $(\tilde{G}, J)$=Ḃ are neutrosophic cubic soft sets in X satisfy the following condition,

$$\min\{miax\{A^+_{F_{e_i}}(x), B^-_{F_{e_i}}(x)\}, \max\{A^-_{F_{e_i}}(x), B^+_{F_{e_i}}(x)\}\} = (\lambda_{F_{e_i}} \wedge \psi_{F_{e_i}})(x)$$
$$= \max\{\min\{A^+_{F_{e_i}}(x), B^-_{F_{e_i}}(x)\}, \min\{A^-_{F_{e_i}}(x), B^+_{F_{e_i}}(x)\}\}$$

For all $x \in X$ .Then $(\tilde{F}, I) \cap_P (\tilde{G}, J)$ of $(\tilde{F}, I)$=Á and $(\tilde{G}, J)$=Ḃ is both F-internal neutrosophic cubic soft set and F-external neutrosophic cubic soft set in X.

Proof:

Consider, $\left(\tilde{F},I\right)\cap_P\left(\tilde{G},J\right)=\left(\tilde{H},C\right)$ where $C=I\cap J$

$$\tilde{H}(e_i)=\tilde{F}(e_i) \quad if \quad e_i\in I-J$$
$$=\tilde{G}(e_i) \quad if \quad e_i\in J-I$$
$$=\tilde{F}(e_i)\wedge_P\tilde{G}(e_i) \quad if \quad e_i\in I\cap J.$$

Where $\tilde{F}(e_i)\wedge_P\tilde{G}(e_i)$ is defined as,

$$\tilde{F}(e_i)\wedge_P\tilde{G}(e_i)=\left\{<x, r\min\left\{A_{Fe_i}(x),B_{Fe_i}(x)\right\},(\lambda_{Fe_i}(x)\wedge\psi_{Fe_i}(x))>: x\in X\right\}, e_i\in I\cap J.$$

Take, $\alpha_{e_i}=\min\left\{\max\left\{A^+_{F_{e_i}}(x),B^-_{F_{e_i}}(x)\right\},\max\left\{A^-_{F_{e_i}}(x),B^+_{F_{e_i}}(x)\right\}\right\}$

$\beta_{e_i}=\max\left\{\min\left\{A^+_{F_{e_i}}(x),B^-_{F_{e_i}}(x)\right\},\min\left\{A^-_{F_{e_i}}(x),B^+_{F_{e_i}}(x)\right\}\right\}$

Then $\alpha_{e_i}=A^-_{F_{e_i}}(x),\ \alpha_{e_i}=B^-_{F_{e_i}}(x),\alpha_{e_i}=A^+_{F_{e_i}}(x)\, or\, \alpha_{e_i}=B^+_{F_{e_i}}(x)$

It is possible to consider, the cases $\alpha_{e_i}=A^-_{F_{e_i}}(x)$ and $\alpha_{e_i}=A^+_{F_{e_i}}(x)$ only because the remaining cases are similar to these cases, if $\alpha_{e_i}=A^-_{F_{e_i}}(x)$,then, $B^-_{F_{e_i}}(x)\le B^+_{F_{e_i}}(x)\le A^-_{F_{e_i}}(x)\le A^+_{F_{e_i}}(x)$

Thus, $\beta_{e_i}=B^+_{F_{e_i}}(x)$, and so, $A^-_{F_{e_i}}(x)=\alpha_{e_i}=(\lambda_{F_{e_i}}\wedge\psi_{F_{e_i}})(x)=\beta_{e_i}=B^+_{F_{e_i}}(x)$

Hence, $B^-_{F_{e_i}}(x)\le B^+_{F_{e_i}}(x)=(\lambda_{F_{e_i}}\wedge\psi_{F_{e_i}})(x)=A^-_{F_{e_i}}(x)\le A^+_{F_{e_i}}(x)$

Which implies that, $(\lambda_{F_{e_i}}\wedge\psi_{F_{e_i}})(x)=B^+_{F_{e_i}}(x)=(A_{F_{e_i}}\cap B_{F_{e_i}})^+(x)$

Hence, $(\lambda_{F_{e_i}}\wedge\psi_{F_{e_i}})(x)\notin((A_{F_{e_i}}\cap B_{F_{e_i}})^-(x),(A_{F_{e_i}}\cap B_{F_{e_i}})^+(x))$

And $(A_{F_{e_i}}\cap B_{F_{e_i}})^-(x)\le(\lambda_{F_{e_i}}\wedge\psi_{F_{e_i}})(x)\le(A_{F_{e_i}}\cap B_{F_{e_i}})^+(x)$

If $\alpha_{e_i}=A^+_{F_{e_i}}(x)$, then, $B^-_{F_{e_i}}(x)\le A^+_{F_{e_i}}(x)\le B^+_{F_{e_i}}(x)$ and thus, $(\lambda_{F_{e_i}}\wedge\psi_{F_{e_i}})(x)=A^+_{F_{e_i}}(x)=(A_{F_{e_i}}\cap B_{F_{e_i}})^+(x)$

Hence, $(\lambda_{F_{e_i}}\wedge\psi_{F_{e_i}})(x)\notin((A_{F_{e_i}}\cap B_{F_{e_i}})^-(x),(A_{F_{e_i}}\cap B_{F_{e_i}})^+(x))$

And $(A_{F_{e_i}}\cap B_{F_{e_i}})^-(x)\le(\lambda_{F_{e_i}}\wedge\psi_{F_{e_i}})(x)\le(A_{F_{e_i}}\cap B_{F_{e_i}})^+(x)$

Consequently,
$\left(\tilde{F},I\right)\cap_P\left(\tilde{G},J\right)$of $(\tilde{F},I)$=Å and $(\tilde{G},J)$=Ḃ is both F-internal neutrosophic cubic soft set and F-external neutrosophic cubic soft set in X.

Theorem: 2.14

For any T-external neutrosophic cubic soft sets $(\tilde{F},I)$=Å and $(\tilde{G},J)$=Ḃ in X, if $(\tilde{F},I)^*$=Å* and $(\tilde{G},J)^*$ =Ḃ* are T-internal neutrosophic cubic soft sets in X ,then $\left(\tilde{F},I\right)\cup_P\left(\tilde{G},J\right)$of $(\tilde{F},I)$=Å and $(\tilde{G},J)$=Ḃ is a T-internal neutrosophic cubic soft sets in X.

Proof:

Consider, $\left(\tilde{F},I\right)\cup_P\left(\tilde{G},J\right)=\left(\tilde{H},C\right)$ where $C=I\cup J$

$$\tilde{H}(e_i)=\tilde{F}(e_i) \quad if \quad e_i\in I-J$$
$$=\tilde{G}(e_i) \quad if \quad e_i\in J-I$$
$$=\tilde{F}(e_i)\vee_P\tilde{G}(e_i) \quad if \quad e_i\in I\cap J.$$

Where $\tilde{F}(e_i)\vee_P\tilde{G}(e_i)$ is defined as,

$$\tilde{F}(e_i)\vee_P\tilde{G}(e_i)=\left\{<x, r\max\left\{A_{Fe_i}(x),B_{Fe_i}(x)\right\},(\lambda_{Fe_i}(x)\vee\psi_{Fe_i}(x))>: x\in X\right\}, e_i\in I\cap J.$$

Assume that $(\tilde{F},I)^*$=Å* and $(\tilde{G},J)^*$=Ḃ* are T-internal neutrosophic cubic soft sets in X for any T-external neutrosophic cubic soft sets $(\tilde{F},I)$=Å and $(\tilde{G},J)$=Ḃ in X. Then,

$\lambda_{T_{e_i}}(x)\notin(A^-_{T_{e_i}}(x),A^+_{T_{e_i}}(x)),\ \psi_{T_{e_i}}(x)\notin(A^-_{T_{e_i}}(x),A^+_{T_{e_i}}(x)),$

$B^-_{T_{e_i}}(x)\le\lambda_{T_{e_i}}(x)\le B^+_{T_{e_i}}(x),\ A^-_{T_{e_i}}(x)\le\psi_{T_{e_i}}(x)\le A^+_{T_{e_i}}(x),$

We now consider the following case,

1) $\lambda_{T_{e_i}}(x)\le A^-_{T_{e_i}}(x)\le\psi_{T_{e_i}}(x)\le A^+_{T_{e_i}}(x),$ and $\psi_{T_{e_i}}(x)\le B^-_{T_{e_i}}(x)\le\lambda_{T_{e_i}}(x)\le B^+_{T_{e_i}}(x),$

2) $A^-_{T_{e_i}}(x)\le\psi_{T_{e_i}}(x)\le A^+_{T_{e_i}}(x)\le\lambda_{T_{e_i}}(x),$ and $B^-_{T_{e_i}}(x)\le\lambda_{T_{e_i}}(x)\le B^+_{T_{e_i}}(x)\le\psi_{T_{e_i}}(x),$

3) $\lambda_{T_{e_i}}(x)\le A^-_{T_{e_i}}(x)\le\psi_{T_{e_i}}(x)\le A^+_{T_{e_i}}(x),$ and $B^-_{T_{e_i}}(x)\le\lambda_{T_{e_i}}(x)\le B^+_{T_{e_i}}(x)\le\psi_{T_{e_i}}(x),$

4) $A^-_{T_{e_i}}(x)\le\psi_{T_{e_i}}(x)\le A^+_{T_{e_i}}(x)\le\lambda_{T_{e_i}}(x),$ and $\psi_{T_{e_i}}(x)\le B^-_{T_{e_i}}(x)\le\lambda_{T_{e_i}}(x)\le B^+_{T_{e_i}}(x),$

First case implies that, $\psi_{T_{e_i}}(x) = A^-_{T_{e_i}}(x) = \lambda_{T_{e_i}}(x) = B^-_{T_{e_i}}(x)$,

Since, $(\tilde{F}, I)^* = \mathring{A}^*$ and $(\tilde{G}, J)^* = \dot{B}^*$ are T-internal neutrosophic cubic soft sets in X, we have $\psi_{T_{e_i}}(x) \le A^+_{T_{e_i}}(x)$ and $\lambda_{T_{e_i}}(x) \le B^+_{T_{e_i}}(x)$. It follows that,

$$
\begin{aligned}
(A_{T_{e_i}} \cup B_{T_{e_i}})^-(x) &= \max\{A^-_{T_{e_i}}(x), B^-_{T_{e_i}}(x)\} = (\lambda_{T_{e_i}}, \psi_{T_{e_i}})(x) \\
&\le \max\{A^+_{T_{e_i}}(x), B^+_{T_{e_i}}(x)\} \\
&= (A_{T_{e_i}} \cup B_{T_{e_i}})^+(x) \quad \forall x \in X.
\end{aligned}
$$

Therefore P-union $(\tilde{F}, I) \cup_P (\tilde{G}, J)$ is a T-internal neutrosophic cubic soft set in X. We can prove the other cases by the similar to the first cases.

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