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DOMINATION IN OPERATION ON BIPOLAR FUZZY GRAPHS

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Abstract:
In this paper, we obtain the bounds of the domination number in operations on bipolar fuzzy graphs like join, Cartesian product, composition, cross product and strong product.

Key Words: Bipolar fuzzy graphs, Domination & Domination Number

1. Introduction:

The first definition of fuzzy graphs was proposed by Kafmann, from the fuzzy relations introduced by Zadeh. Although Rosenfeld introduced another elaborated definition, including fuzzy vertex and fuzzy edges, and several fuzzy analogs of graph theoretic concepts such as paths, cycles, connectedness and etc. The concept of domination in fuzzy graphs was investigated by A. Somasundaram, S. Somasundaram and A. Somasundaram present the concepts of independent domination, total domination, connected domination of fuzzy graphs. Akram introduced the concept of bipolar fuzzy graphs, he discussed the concept of isomorphism of these graphs, and investigated some of their important properties, also defined some operations on bipolar fuzzy graphs. In this paper, we obtain the bonds of the domination number in operations on bipolar fuzzy graphs like join, Cartesian product, composition, cross product and strong product.

2. Preliminaries:

In this section, some basic definitions related to bipolar fuzzy are given.

A bipolar fuzzy graph (BFG) is of the form G = (V, E) where

i) $V=\{v_1,v_2,\ldots v_n\}$ such that $\mu_1^+ : X \to [0,1]$ and $\mu_1^- : X \to [-1,0]$

ii) $E \subset V\times V$ where $\mu_2^+ : V\times V \to [0,1]$ and $\mu_2^- : V\times V \to [-1,0]$

Such that $\mu_{2_{ij}}^+ = \mu_2^+(v_iv_j) \le \min(\mu_1^+(v_i),\mu_1^+(v_j))$ and

$\mu_{2_{ij}}^- = \mu_2^-(v_iv_j) \le \max(\mu_1^-(v_i),\mu_1^-(v_j))$ for all $(v_i, v_j)\in E$.

A bipolar fuzzy graph (BFG) G = (V, E) is called strong if $\mu_2^+(v_i,v_j)=\max(\mu_1^+(v_i),\mu_1^+(v_j))$ and $\mu_2^-(v_i,v_j)=\min(\mu_1^-(v_i),\mu_1^-(v_j))$ for all $(v_i, v_j)\in E$.

A bipolar fuzzy graph (BFG) G = (V, E) is called complete if $\mu_2^+(v_i,v_j)=\min(\mu_1^+(v_i),\mu_1^+(v_j))$ and $\mu_2^-(v_i,v_j)=\max(\mu_1^-(v_i),\mu_1^-(v_j))$ for all $(v_i, v_j)\in V$.

Let G = (V, E) be a bipolar fuzzy graph. Then the cardinality of G is defined to be

$$|G| = \sum_{v_i\in V}\left(\frac{1+\mu_1^+(v_i)+\mu_1^-(v_i)}{2}\right) + \sum_{(v_i,v_j)\in E}\left(\frac{1+\mu_2^+(v_iv_j)+\mu_2^-(v_iv_j)}{2}\right)$$

Let G = (V, E) be a bipolar fuzzy graph. Then the vertex cardinality of G is defined by

$$|V| = \sum_{v_i\in V}\left(\frac{1+\mu_1^+(v_i)+\mu_2^-(v_j)}{2}\right) \text{ for all } v_j \in V .$$

The number of vertices (the cardinality of V) is called the order of a BFG and is defined by

$$O(G) = \sum_{v_i\in V}\left(\frac{1+\mu_1^+(v_i)+\mu_1^-(v_i)}{2}\right)$$

Let G = (V, E) be a bipolar fuzzy graph. Then the edge cardinality of G is defined by

$$|E| = \sum_{(v_i,v_j)\in E}\left(\frac{1+\mu_2^+(v_iv_j)+\mu_2^-(v_iv_j)}{2}\right).$$

The number of edges (the cardinality of E) is called the size of a BFG and is defined by

$$S(G) = \sum_{(v_i,v_j)\in E}\left(\frac{1+\mu_2^+(v_iv_j)+\mu_2^-(v_iv_j)}{2}\right) \text{ for all } (v_iv_j) \in E .$$

The degree of a vertex v in a BFG, G(V, E) is defined to be sum of the cardinality of strong arcs incident at v .It is denoted by $d_G(v)$.The minimum degree of G is $\delta(G)=\min\{d_G(v)/v\in V\}$ The maximum degree of g is $\Delta(G) = \max\{d_G(v)/v \in V\}$.

The two vertices v_i and v_j are said to be effective neighbours in BFG if either one of the following conditions holds

i) $\mu_2^+(v_iv_j) > 0 \; and \; \mu_2^-(v_iv_j) = 0$

ii) $\mu_2^+(v_iv_j) = 0 \; and \; \mu_2^-(v_iv_j) < 0$

iii) $\mu_2^+(v_iv_j) > 0 \; and \; \mu_2^-(v_iv_j) < 0, (v_i, v_j) \in E$

The complement of a BFG, G = (V, E) is a BFG $\overline{G} = (\overline{V}, \overline{E})$, where

i) $\overline{V} = V$

ii) $\overline{\mu_{1i}^+} = \mu_{1i}^+$ and $\overline{\mu_{1i}^-} = \mu_{1i}^-$ for all i = 1, 2, ...n.

iii) $\overline{\mu_{2_{ij}}^+} = \min(\mu_{1_i}^+, \mu_{1_j}^+) - \mu_{2_{ij}}^+$ and

$\overline{\mu_{2_{ij}}^-} = \max(\mu_{1i}^-, \mu_{1j}^-) - \mu_{2ij}^-$ for all i, j = 1, 2, ..n.

An edge *(u, v)* is said to be strong edge in BFG, G = (V, E) if $\mu_2^+(u,v) \geq (\mu_2^+)^\infty(u,v), \mu_2^-(u,v) \leq (\mu_2^-)^\infty(u,v)$

Where $\mu_2^+(u,v) = \max\{(\mu_2^+)^k(u,v)/k = 1,2,3,..n\} \; and$

$\mu_2^-(u,v) = \min\{(\mu_2^-)^k(u,v)/k = 1,2,3,..n\}$

A vertex *u* be a vertex in BFG, G = (V, E) then N(*u*) = {*v*, *v*$\in$*V* and *(u, v)* is a strong arc in G} is called neighbourhood of *u* in G. A vertex *u*$\in$*V* of a BFG, G = (V, E) is said to be isolated vertex if $\mu_2^+(u,v) = 0 \, and \; \mu_2^-(u,v) = 0 \; for \; all \; v \in V, u \neq v$ That is N(*u*) = 0. Thus an isolated vertex does not dominate any other vertex of G.Let G = (V, E) be a BFG on V. Let *u*, *v*$\in$*V* , we say that *u* dominates *v* in G if there exists a strong edge between them.

Note:

i) For any *u*, *v*$\in$*V* of *u* dominates *v* then *v* dominates *u* and hence domination is a symmetric relation on *V*.

ii) For any *v*$\in$*V*, *N(v)* is precisely the set of all vertices in *V* which are dominated by *v*.

iii) If $\mu_2^+(u,v) < (\mu_2^+)^\infty(u,v) \; and \; \mu_2^-(u,v) > (\mu_2^-)^\infty(u,v)$ for all *u*, *v*$\in$*V*. Then the dominating set of G is *V*.

A subset S of *V* is called dominating set in G if for every *v*$\in$*V- S*, there exist *u*$\in$*S* such that *u* dominates *v*. A dominating set S of a BFG is said to be minimal dominating set if no proper subset of S is a dominating set. The Minimum cardinality among all minimal dominating set is called the lower domination number of G and is denoted by γ_{bf}(G). The Maximum cardinality among all minimal dominating set is called the upper domination number of G and is denoted by Γ_{bf}(G).

3. Main Results:

Definition 3.1:

Let $A_1 = (\mu_{A_1}^+, \mu_{A_1}^-)$ and $A_2 = (\mu_{A_2}^+, \mu_{A_2}^-)$ be bipolar fuzzy subsets of V_1 *and* V_2 in which $V_1 \cap V_2 = \phi$ and let $B_1 = (\mu_{B_1}^+, \mu_{B_1}^-)$ and $B_2 = (\mu_{B2}^+, \mu_{B_2}^-)$ be bipolar fuzzy subsets of $V_1 \times V_2 \; and \; V_2 \times V_1$ respectively. Then we denote the join of two bipolar fuzzy graphs G_1 and G_2 by $G_1 + G_2 = (A_1 + A_2, \, B_1 + B_2)$ and defined as follows $\begin{cases} (\mu_{A_1}^+ + \mu_{A_2}^+)(x) = \max(\mu_{A_1}^+(x), \mu_{A_2}^+(x)) \\ (\mu_{A_1}^- + \mu_{A_2}^-)(x) = \min(\mu_{A_1}^-(x), \, \mu_{A_2}^-(x)) \; if \;\; x \in V_1 \cup V_2 \end{cases}$

$$\begin{cases} (\mu_{B_1}^+ + \mu_{B_2}^+)(xy) = \max(\mu_{B_1}^+(xy), \, \mu_{B_2}^+(xy)) \\ (\mu_{B_1}^- + \mu_{B_2}^-)(xy) = \min(\mu_{B_1}^-(xy), \, \mu_{B_2}^-(xy)) \; if \;\; xy \in E_1 \cap E_2 \end{cases} \quad \begin{cases} (\mu_{B_1}^+ + \mu_{B_2}^+)(xy) = \min(\mu_{B_1}^+(x), \, \mu_{B_1}^+(y)) \\ (\mu_{B_1}^- + \mu_{B_2}^-)(xy) = \max(\mu_{B_1}^-(x), \, \mu_{B_2}^-(y)) \; if \;\; xy \in E_1^/ \end{cases}$$

Where $E_1^/$ is the set of all edges joining the nodes of V_1 and V_2.

Theorem 3.1:

Let $G_1 and \, G_2$ be a bipolar fuzzy graphs and $D_1 and \, D_2$ be the minimum dominating set of $G_1 \; and \; G_2$ respectively. Then $\gamma(G_1 + G_2) = \min(|D_1|, |D_2|) = \min(\gamma(G_1), \, \gamma(G_2))$

Proof:

First we prove $D_1 and \, D_2$ be the dominating set of $G_1 + G_2$. If $y \in G_1$, obviously is dominating by $x \in D_1$, since D_1 is the dominating set of G_1.

If $y \in G_2$.Now we prove $x \in D_1$ dominates y $x \in D_1 \subset G_1$ *and* $y \in G_2$.

$$(\mu_{B_1}{}^{+} \cup \mu_{B_2}{}^{+})(xy) = \min(\mu_{B_1}{}^{+}(x), \mu_{B_2}{}^{+}(y))$$

$$(\mu_{B_1}{}^{-} \cup \mu_{B_2}{}^{-})(xy) = \max(\mu_{B_1}{}^{-}(x), \mu_{B_2}{}^{-}(y))$$

Clearly *x* dominates y. Therefore D_1 is the dominating set of $G_1 + G_2$.

We assume D_1 is not a minimum dominating set of $G_1 + G_2$. There exist $x \in D_1$ such that $D_1 - x$ is a dominating set of $G_1 + G_2$. Therefore $x \in G_1 + G_2$, there exist $y \in D_1$ such that

$$(\mu_{B_1}{}^{+} \cup \mu_{B_2}{}^{+})(xy) = \min(\mu_{B_1}{}^{+}(x), \mu_{B_2}{}^{+}(y))$$

$$(\mu_{B_1}{}^{-} \cup \mu_{B_2}{}^{-})(xy) = \max(\mu_{B_1}{}^{-}(x), \mu_{B_2}{}^{-}(y))$$

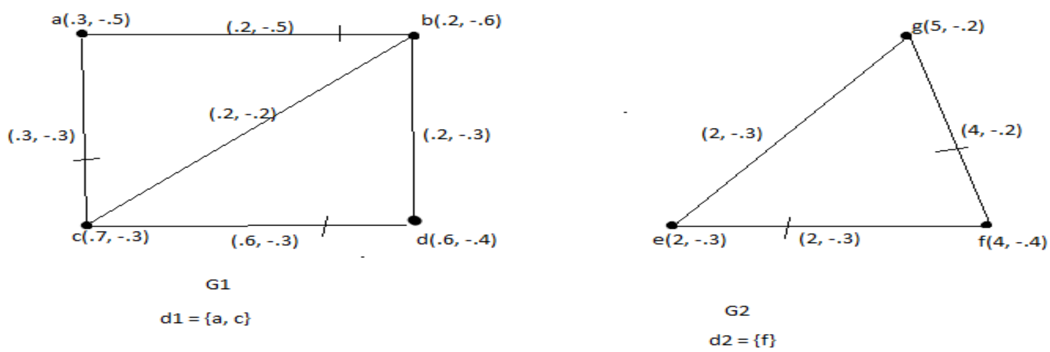
This implies $D_1 - x$ is the dominating set of G_1. This is contradicting to D_1 is minimum.

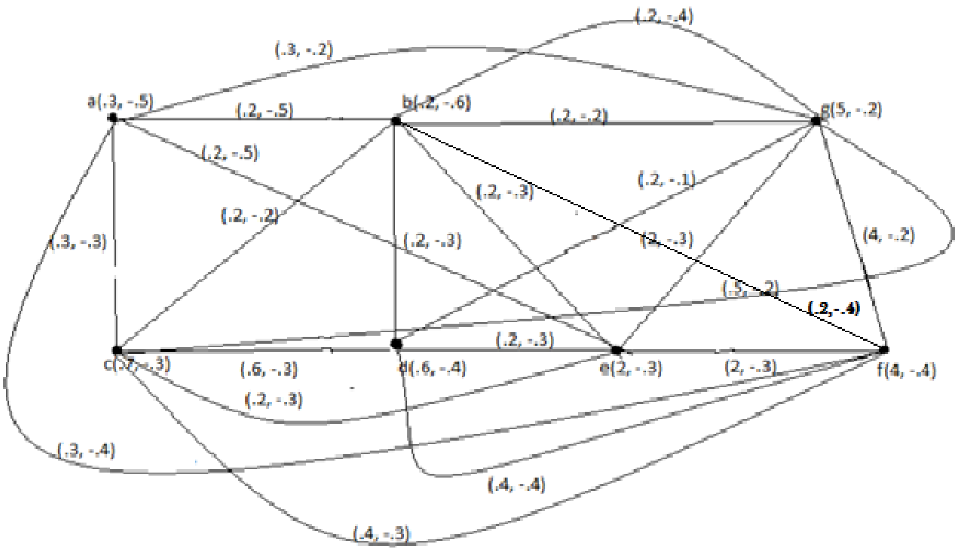
Similarly we prove D_2 is a minimum dominating set of $G_1 + G_2$. Let D be the minimum dominating set of $G_1 + G_2$.

$$|D| = |D_1 + D_2|$$

$$\gamma(G_1 + G_2) = \min(\gamma(G_1), \gamma(G_2))$$

Example 3.1:





G$_1$+G$_2$

Figure 3.1

In figure 3.1 the minimum dominating set of G_1 and G_2 are D_1={a,c} and D_2={f} respectively. The minimum dominating set of G_1+G_2 is {f}.

Definition 3.2:

The Cartesian product G1 ×G2 is the pair (A,B) of bipolar fuzzy sets defined on the Cartesian product $G_1^* \times G_2^*$ such that

$$\begin{cases} \mu_A^+(x_1,x_2) = \min(\mu_{A_1}^+(x_1), \mu_{A_2}^+(x_2)) \\ \mu_A^-(x_1,x_2) = \max(\mu_{A_1}^-(x_1), \mu_{A_2}^-(x_2)) \; for\ all\ (x_1,x_2) \in V_1 \times V_2 \end{cases} \quad \begin{cases} \mu_B^+(x,x_2)(x,y_2) = \min(\mu_{A_1}^+(x), \mu_{B_2}^+(x_2y_2)) \\ \mu_B^-(x,x_2)(x,y_2) = \max(\mu_{A_1}^-(x), \mu_{B_2}^-(x_2y_2)) \; for\ all\ x_2y_2 \in E_2 \end{cases}$$

$$\begin{cases} \mu_B^+(x_1,z)(y_1,z) = \min(\mu_{B_1}^+(x_1y_1), \mu_{A_2}^+(z)) \\ \mu_B^-(x_1,z)(y_1,z) = \max(\mu_{B_1}^-(x_1y_1), \mu_{A_2}^-(z)) \; or\ all\ x_1y_1 \in E_1 \end{cases}$$

Theorem 3.2:

Let D_1 and D_2 be a minimum dominating set of bipolar fuzzy graph $G_1 \,\&\, G_2$ respectively. Then

$$\gamma(G_1 \times G_2) = \min\{|D_1 \times V_2|, |V_1 \times D_2|\}.$$

Proof:

Now we prove $D_1 \times V_2$ is a dominating set of $G_1 \times G_2$. Let $(u_1,u_2) \notin D_1 \times V_2$. Hence $u_1 \notin D_1$. Let D_1 is a dominating set of G_1, there exist $V_1 \in D$ such that

$$\mu_{B_1}^+(u_1,v_1) = \min(\mu_{A_1}^+(u_1),\ \mu_{A_1}^+(v_1))$$

$$\mu_{B_1}^-(u_1,v_1) = \max(\mu_{A_1}^-(u_1),\ \mu_{A_1}^-(v_1))$$

If $(v_1u_2) \in D_1 \times V_2$,

$$\begin{aligned} \mu_B^+((u_1,u_2)(v_1,u_2)) &= \min(\mu_{B_1}^+(u_1v_1),\ \mu_{A_2}^+(u_2)) \quad \forall u_2 \in v_2 \\ &= \min((\mu_{A_1}^+(u_1) \wedge \mu_{A_1}^+(v_1)), \mu_{A_2}^+(u_2)) \\ &= \min(\mu_{A_1}^+(u_1) \wedge \mu_{A_1}^+(v_1) \wedge \mu_{A_2}^+(u_2)) \\ &= \min(\mu_{A_1}^+(u_1) \wedge \mu_{A_2}^+(u_2)) \min(\mu_{A_1}^+(v_1) \wedge \mu_{A_2}^+(u_2)) \\ &= \mu_A^+(u_1u_2) \wedge \mu_A^+(v_1u_2) \end{aligned}$$

$$\begin{aligned} \mu_B^-((u_1,u_2)(v_1,u_2)) &= \max(\mu_{B_1}^-(u_1v_1),\ \mu_{A_2}^-(u_2)) \\ &= \max((\mu_{A_1}^-(u_1) \vee \mu_{A_1}^-(v_1)), \mu_{A_2}^-(u_2)) \\ &= \max(\mu_{A_1}^-(u_1) \vee \mu_{A_1}^-(v_1) \wedge \mu_{A_2}^-(u_2)) \\ &= \max(\mu_{A_1}^-(u_1) \vee \mu_{A_2}^-(u_2)) \max(\mu_{A_1}^-(v_1) \vee \mu_{A_2}^-(u_2)) \\ &= \mu_B^-(u_1u_2) \vee \mu_B^-(v_1,u_2) \end{aligned}$$

This implies every $(u_1,u_2) \notin D_1 \times V_2$ there exist a vertex $(u_1,u_2) \notin D_1 \times V_2$ such that there **is** a strong arc between (u_1,u_2) and (v_1,u_2) .Therefore $D_1 \times V_2$ is a dominating set of $G_1 \times G_2$. Similarly we prove $V_1 \times D_2$ is a dominating set of $G_1 \times G_2$.

Next we prove $D_1 \times V_2$ and $V_1 \times D_2$ is a minimum dominating set of $G_1 \times G_2$ .Suppose $D_1 \times V_2$ is not a minimal. There exist a vertex $(x_1,u_2) \in D_1 \times V_2$ such that $\{(D_1 \times V_2) - (x_1,u_2)\}$ is a dominating set of $G_1 \times G_2$. Let $(u_1,u_2) \in D_1 \times V_2$, (u_1,u_2) dominates (x_1,u_2) .Since $D_1 \times V_2$ is a dominating set of $G_1 \times G_2$. Therefore we get u_1 dominates $x_1 \in D_1$. Clearly $D_1 - x_1$ is a dominating set of G_1 .This is contradict to our assumption D_1 is minimal. Therefore $D_1 \times V_2$ is a minimal dominating set of $G_1 \times G_2$.

Similarly we find $V_1 \times D_2$ is a minimum dominating set of $G_1 \times G_2$.

Therefore $\gamma(G_1 \times G_2) = \min\{|D_1 \times V_2|, |V_1 \times D_2|\}$.

Definition 3.3:

The composition $G_1[G_2]$ is the pair (A, B) of bipolar fuzzy sets defined on the composition $G_1^*[G_2^*]$ such that

$$\begin{cases} \mu_A^+(x_1,x_2) = \min(\mu_{A_1}^+(x_1), \mu_{A_2}^+(x_2)) \\ \mu_A^-(x_1,x_2) = \max(\mu_{A_1}^-(x_1), \mu_{A_2}^-(x_2)) \; for\ all\ (x_1,x_2) \in V_1 \times V_2 \end{cases}$$

$$\begin{cases} \mu_B^+(x,x_2)(x,y_2) = \min(\mu_{A_1}^+(x), \mu_{B_2}^+(x_2y_2)) \\ \mu_B^-(x,x_2)(x,y_2) = \max(\mu_{A_1}^-(x), \mu_{B_2}^-(x_2y_2)) \; for\ all\ x_2y_2 \in E_2 \end{cases} \quad \begin{cases} \mu_B^+(x_1,z)(y_1,z) = \min(\mu_{B_1}^+(x_1y_1), \mu_{A_2}^+(z)) \\ \mu_B^-(x_1,z)(y_1,z) = \max(\mu_{B_1}^-(x_1y_1), \mu_{A_2}^-(z)) \end{cases}$$

$$\begin{cases} \mu_B^+((x_1,x_2)(y_1y_2)) = \min(\mu_{A_2}^+(x_2), \mu_{A_2}^+(y_2), \mu_{B1}^+(x_1y_1)) \\ \mu_B^-((x_1,x_2)(y_1y_2)) = \max(\mu_{A_2}^-(x_2), \mu_{A_2}^+(y_2), \mu_{B1}^-(x_1y_1)) \end{cases}$$

Theorem 3.3:

Let D_1, D_2 are minimum dominating sets of a bipolar fuzzy graphs G_1 *and* G_2 respectively. Then

$$\gamma(G_1 \circ G_2) = |D_1 \times D_2|.$$

Proof:

Let $(u, v) \notin D_1 \times D_2$.

Case (i): $u \notin D_1$ and $v \in D_2$

Let $u \notin D_1$, there exist $u_1 \in D_1$ such that u_1 dominates u. Then

$\mu_{B_1}{}^{+}(u, u_1) = \min(\mu_{A_1}{}^{+}(u), \mu_{A_1}{}^{+}(u_1))$ $\mu_{B_1}{}^{-}(u, u_1) = \max(\mu_{A_1}{}^{-}(u), \mu_{A_1}{}^{-}(u_1))$

Now $(u_1, v) \in D_1 \times D_2$,

$$\begin{aligned}
\mu_B{}^{+}((u,v)(u_1,v)) &= \mu_{B_1}{}^{+}(u,u_1) \wedge \mu_{A_2}{}^{+}(v) \\
&= \min((\mu_{A_1}{}^{+}(u), \mu_{A_1}{}^{+}(u_1)) \wedge \mu_{A_2}{}^{+}(v)) \\
&= \mu_{A_1}{}^{+}(u) \wedge \mu_{A_2}{}^{+}(v) \wedge \mu_{A_1}{}^{+}(u_1) \wedge \mu_{A_2}{}^{+}(v)) \\
&= (\mu_{A_1} \circ \mu_{A_2})^{+}(u,v) \wedge (\mu_{A_1} \circ \mu_{A_2})^{+}(u_1,v)
\end{aligned}$$

$$\begin{aligned}
\mu_B{}^{-}((u,v)(u_1,v_1)) &= \mu_{B_1}{}^{-}(u,u_1) \vee \mu_{A_2}{}^{-}(v) \\
&= \max((\mu_{A_1}{}^{-}(u), \mu_{A_1}{}^{-}(u_1)) \wedge \mu_{A_2}{}^{-}(v)) \\
&= \mu_{A_1}{}^{-}(u) \vee \mu_{A_2}{}^{-}(v) \vee \mu_{A_1}{}^{-}(u_1) \vee \mu_{A_2}{}^{-}(v)) \\
&= (\mu_{A_1} \circ \mu_{A_2})^{-}(u,v) \vee (\mu_{A_1} \circ \mu_{A_2})^{-}(u_1,v)
\end{aligned}$$

Hence (u_1, v) dominates (u, v).

Case (ii): $u \in D_1$ and $v \notin D_2$

Let $v_2 \in D_2$, such that

$$\mu_{B_2}{}^{+}(v, v_2) = \min(\mu_{A_2}{}^{+}(v), \mu_{A_2}{}^{+}(v_2))$$

$$\mu_{B_2}{}^{-}(v, v_2) = \max(\mu_{A_2}{}^{-}(v), \mu_{A_2}{}^{-}(v_2))$$

Now $(u, v_2) \in D_1 \times D_2$,

$$\begin{aligned}
\mu_B{}^{+}((u,v)(u,v_2)) &= \min(\mu_{A_1}{}^{+}(u), \mu_{B_2}{}^{+}(v,v_2)) \\
&= \min((\mu_{A_1}{}^{+}(u), \mu_{A_2}{}^{+}(v)) \wedge \mu_{A_2}{}^{+}(v_2)) \\
&= \mu_{A_1}{}^{+}(u) \wedge \mu_{A_2}{}^{+}(v) \wedge \mu_{A_1}{}^{+}(u) \wedge \mu_{A_2}{}^{+}(v_2)) \\
&= (\mu_{A_1} \circ \mu_{A_2})^{+}(u,v) \wedge (\mu_{A_1} \circ \mu_{A_2})^{+}(u,v_2)
\end{aligned}$$

$$\begin{aligned}
\mu_B{}^{-}((u,v)(u,v_2)) &= \max(\mu_{A_1}{}^{-}(u), \mu_{B_2}{}^{-}(v,v_2)) \\
&= \mu_{A_1}{}^{-}(u) \vee \mu_{A_2}{}^{-}(v)) \vee \mu_{A_2}{}^{-}(v_2)) \\
&= \mu_{A_1}{}^{-}(u) \vee \mu_{A_2}{}^{-}(v) \vee \mu_{A_1}{}^{-}(u) \vee \mu_{A_2}{}^{-}(v_2)) \\
&= (\mu_{A_1} \circ \mu_{A_2})^{-}(u,v) \vee (\mu_{A_1} \circ \mu_{A_2})^{-}(u,v_2)
\end{aligned}$$

Hence (u, v_1) dominates (u, v).

Case (iii): $u \notin D_1$ and $v \notin D_2$

D_1 and D_2 be the minimum dominating sets of G_1 *and* G_2.

Therefore, there exist $u_1 \in D_1$ and $v_2 \in D_2$ such that

$$\mu_{B_1}{}^{+}(u, u_1) = \min(\mu_{A_1}{}^{+}(u), \mu_{A_1}{}^{+}(u_1))$$

$$\mu_{B_1}{}^{-}(u, u_1) = \max(\mu_{A_1}{}^{-}(u), \mu_{A_1}{}^{-}(u_1))$$

and $\mu_{B_2}{}^{+}(v, v_2) = \min(\mu_{A_2}{}^{+}(v), \mu_{A_2}{}^{+}(v_2))$ $\mu_{B_2}{}^{-}(v, v_2) = \max(\mu_{A_2}{}^{-}(v), \mu_{A_2}{}^{-}(v_2))$

Let $(u_1, v_2) \in D_1 \times D_2$,

$$\mu_B^{+}((u,v)(u_1,v_2)) = \min(\mu_{A_2}^{+}(v),\mu_{A_2}^{+}(v_2),\mu_{B_1}^{+}(u,u_1))$$
$$= \min(\mu_{A_2}^{+}(v),\mu_{A_2}^{+}(v_2),\mu_{A_1}^{+}(u)\wedge\mu_{A_1}^{+}(u_1))$$
$$= \mu_{A_1}^{+}(u)\wedge\mu_{A_2}^{+}(v)\wedge\mu_{A_1}^{+}(u)\wedge\mu_{A_2}^{+}(v_2))$$
$$= (\mu_{A_1}\circ\mu_{A_2})^{+}(u,v)\wedge(\mu_{A_1}\circ\mu_{A_2})^{+}(u_1,v_2)$$
$$\mu_B^{-}((u,v)(u_1,v_2)) = \max(\mu_{A_2}^{-}(v),\mu_{A_2}^{-}(v_2),\mu_{B_1}^{-}(u,u_1))$$
$$= \max(\mu_{A_2}^{-}(v),\mu_{A_2}^{-}(v_2),(\mu_{A_1}^{-}(u)\vee\mu_{A_1}^{-}(u_1))$$
$$= \mu_{A_1}^{-}(u)\vee\mu_{A_2}^{-}(v)\vee\mu_{A_1}^{-}(u)\vee\mu_{A_2}^{-}(v_2))$$
$$= (\mu_{A_1}\circ\mu_{A_2})^{-}(u,v)\vee(\mu_{A_1}\circ\mu_{A_2})^{-}(u_1,v_2)$$
$$= \max(\mu_A^{-}(u,v),\mu_A^{-}(u_1,v_2))$$

Therefore we get (u_1,v_2) dominates (u,v) in $G_1\circ G_2$. This implies that $D_1\times D_2$ is a dominating set of $G_1\circ G_2$.

Now we prove $D_1\times D_2$ is minimum. Let $(x_1,x_2)\in D_1\times D_2$, $x_1\in D_1$ and $x_2\in D_2$. By our assumption D_1 and D_2 are minimal dominating set of G_1 and G_2 respectively. Therefore D_1-x_1 and D_2-x_2 are not a dominating set.Clearly we get $(D_1\times D_2)-(x_1\times x_2)$ is not a minimal dominating set.This implies that $D_1\times D_2$ is minimal dominating set of $G_1\circ G_2$.

Therefore we get $\gamma(G_1\circ G_2)=|D_1\times D_2|$..

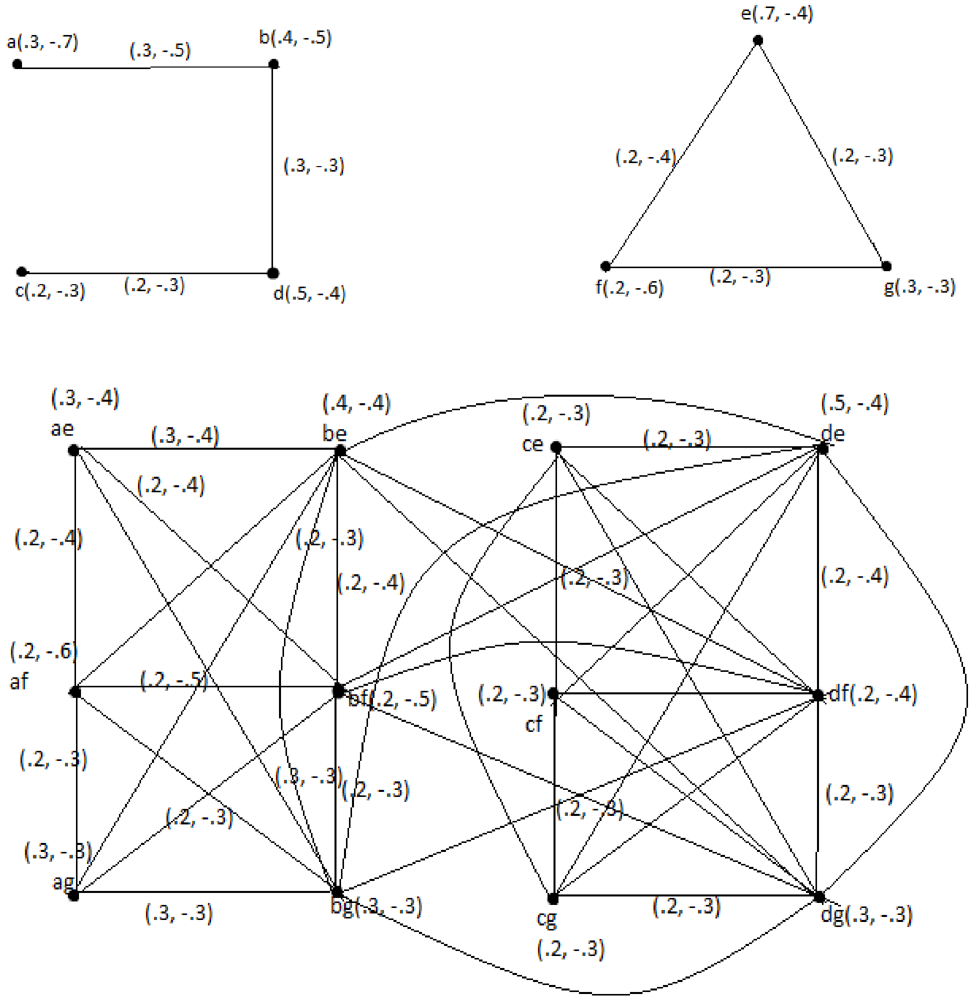


Figure 3.2

In figure 3.2 the minimum dominating set of G_1 and G_2 are D_1={a,c} and D_2={f} respectively. The minimum dominating set of $G_1\circ G_2$ is {af,cf}.

Definition 3.4:

The cross product of $G_1 * G_2$ is the pair (A, B) of bipolar fuzzy sets defined on the cross product $G_1^* * G_2^*$ such that

$$\begin{cases} \mu_A^+(x_1,x_2) = \min(\mu_{A_1}^+(x_1), \mu_{A_2}^+(x_2)) \\ \mu_A^-(x_1,x_2) = \max(\mu_{A_1}^-(x_1), \mu_{A_2}^-(x_2)) \end{cases}$$

$$\begin{cases} \mu_{\mathbf{B}}^+(x_1,x_2)(y_1,y_2) = \min(\mu_{B_1}^+(x_1y_1), \mu_{B_2}^+(x_2y_2)) \\ \mu_B^-(x_1,x_2)(y_1,y_2) = \max(\mu_{B_1}^-(x_1y_1), \mu_{B_2}^-(x_2y_2)) \end{cases}$$

$$\textit{for all } x_1y_1 \in E_1 \ \& \ x_2y_2 \in E_2$$

Theorem 3.4:

Let D_1 and D_2 be the minimum dominating set of bipolar fuzzy graph $G_1 \textit{and } G_2$ respectively. Then

$$\gamma_{bf}(G_1 * G_2) = \min\{|D_1 \times V_2|, |V_1 \times D_2|\}$$

Proof:

Let $(u_1,u_2) \notin D_1 \times V_2$. Hence $u_1 \notin D_1$.

Since D_1 is a dominating set of G_1, there exist a vertex $V_1 \in D_1$ such that

$$\mu_{B_1}^+(u_1,v_1) = \min(\mu_{A_1}^+(u_1),\ \mu_{A_1}^+(v_1))$$

$$\mu_{B_1}^-(u_1,v_1) = \max(\mu_{A_1}^-(u_1),\ \mu_{A_1}^-(v_1))$$

Let $(v_1v_2) \in D_1 \times V_2$ such that,

$$\mu_A^+(u_1u_2) = \min(\mu_{A_1}^+(u_1),\ \mu_{A_2}^+(u_2))$$

$$\mu_A^-(u_1u_2) = \max(\mu_{A_1}^-(u_1),\ \mu_{A_2}^-(u_2))$$

and $\mu_A^+(v_1v_2) = \min(\mu_{A_1}^+(v_1),\ \mu_{A_2}^+(v_2))$

$$\mu_A^-(v_1v_2) = \max(\mu_{A_1}^-(v_1),\ \mu_{A_2}^-(v_2))$$

Now

$$\begin{aligned} \mu_B^+((u_1u_2)(v_1u_2)) &= \mu_A^+(u_1u_2) \wedge \mu_A^+(v_1v_2) \\ &= (\mu_{A_1}^+(u_1) \wedge \mu_{A_2}^+(u_2) \wedge \mu_{A_1}^+(v_1)) \wedge \mu_{A_2}^+(v_2)) \\ &= (\mu_{A_1}^+(u_1) \wedge \mu_{A_1}^+(v_1) \wedge (\mu_{A_2}^+(u_2) \wedge \mu_{A_2}^+(v_2)) \\ &= \mu_{B_1}^+(u_1v_1) \wedge \mu_{B_2}^+(u_2v_2) \end{aligned}$$

$$\begin{aligned} \mu_B^-((u_1u_2)(v_1u_2)) &= \mu_A^-(u_1u_2) \vee \mu_A^-(v_1v_2) \\ &= (\mu_{A_1}^-(u_1) \vee \mu_{A_2}^-(u_2) \vee \mu_{A_1}^-(v_1)) \vee \mu_{A_2}^-(v_2)) \\ &= (\mu_{A_1}^-(u_1) \vee \mu_{A_1}^-(v_1) \vee (\mu_{A_2}^-(u_2) \vee \mu_{A_2} - (v_2)) \\ &= \mu_{B_1}^-(u_1v_1) \vee \mu_{B_2}^-(u_2v_2) \end{aligned}$$

This implies there is strong arc between (u_1,u_2) and (v_1,v_2).

Therefore (u_1,u_2) is a dominating by (v_1,v_2).

This implies $D_1 \times V_2$ is a dominating by $(G_1 * G_2)$.

Similarly we prove $V_1 \times V_2$ is a dominating by $(G_1 * G_2)$.

Hence $\gamma_{bf}(G_1 * G_2) = \min\{|D_1 \times V_2|, |V_1 \times D_2|\}$.

Definition 3.6:

The strong product $G_1 \otimes G_2$ of G_1 is the pair (A, B) of bipolar fuzzy sets defined on the strong product $G_1^* \otimes G_2^*$ such that

$$\begin{cases} \mu_A^+(x_1,x_2) = \min(\mu_{A_1}^+(x_1), \mu_{A_2}^+(x_2)) \\ \mu_A^-(x_1,x_2) = \max(\mu_{A_1}^-(x_1), \mu_{A_2}^-(x_2)) \quad \textit{for all } (x_1,x_2) \in V_1 \times V_2 \end{cases}$$

$$\begin{cases} \mu_{\mathbf{B}}^+(x,x_2)(x,y_2) = \min(\mu_{A_1}^+(x), \mu_{B_2}^+(x_2y_2)) \\ \mu_B^-(x,x_2)(x,y_2) = \max(\mu_{A_1}^-(x), \mu_{B_2}^-(x_2y_2)) \quad \textit{for all } x \in V_1 \,\&\, x_2y_2 \in E_2 \end{cases}$$

$$\begin{cases} \mu_{\mathbf{B}}^+(x_1,z)(y_1,z) = \min(\mu_{B_1}^+(x_1y_1), \mu_{A_2}^+(z)) \\ \mu_B^-(x_1,z)(y_1,z) = \max(\mu_{B_1}^-(x_1y_1), \mu_{A_2}^-(z)) \quad \textit{for all } z \in V_2 \,\&\, \textit{for all } x_1y_1 \in E_1 \end{cases}$$

$$\begin{cases} \mu_{\mathbf{B}}^+((x_1x_2)(y_1y_2)) = \min(\mu_{B_1}^+(x_1y_1), \mu_{\mathbf{B}_1}^+(x_2y_2)) \\ \mu_B^-((x_1x_2)(y_1y_2)) = \max(\mu_{B_1}^-(x_1y_1), \mu_{\mathbf{B}2}^-(x_2y_2)) \quad \textit{for all } (x_1y_1) \in E_1 \,\&\, (x_2y_2) \in E_2 \end{cases}$$

Theorem 3.5:

Let D_1, D_2 are minimum dominating sets of a bipolar fuzzy graph G_1 *and* G_2 respectively. Then $\gamma(G_1 \otimes G_2) = |D_1 \times D_2|$.

Proof:

Let $(u\ ,v\ ) \notin D_1 \times D_2$.

Case (i): $u \notin D_1$ and $v \in D_2$

Let $u \notin D_1$, there exist $u_1 \in D_1$ such that u_1 dominates *u*. Then

$$\mu_{B_1}{}^{+}(u\ ,u_1) = \min(\mu_{A_1}{}^{+}(u\ ), \mu_{A_1}{}^{+}(u_1))$$

$$\mu_{B_1}{}^{-}(u\ ,u_1) = \max(\mu_{A_1}{}^{-}(u), \mu_{A_1}{}^{-}(u_1))$$

Now $(u_1, v) \in D_1 \times D_2$,

$$\begin{aligned}\mu_B{}^{+}((u,v)(u_1,v)) &= \mu_{B_1}{}^{+}(u,u_1) \wedge \mu_{A_2}{}^{+}(v)\\ &= \min((\mu_{A_1}{}^{+}(u), \mu_{A_1}{}^{+}(u_1)) \wedge \mu_{A_2}{}^{+}(v))\\ &= \mu_{A_1}{}^{+}(u) \wedge \mu_{A_2}{}^{+}(v) \wedge \mu_{A_1}{}^{+}(u_1) \wedge \mu_{A_2}{}^{+}(v))\\ &= (\mu_{A_1} \otimes \mu_{A_2})^{+}(u,v) \wedge (\mu_{A_1} \otimes \mu_{A_2})^{+}(u_1,v)\end{aligned}$$

$$\begin{aligned}\mu_B{}^{-}((u,v)(u_1,v)) &= \mu_{B_1}{}^{-}(u,u_1) \vee \mu_{A_2}{}^{-}(v)\\ &= \max((\mu_{A_1}{}^{-}(u), \mu_{A_1}{}^{-}(u_1)) \wedge \mu_{A_2}{}^{-}(v))\\ &= \mu_{A_1}{}^{-}(u) \vee \mu_{A_2}{}^{-}(v) \vee \mu_{A_1}{}^{-}(u_1) \vee \mu_{A_2}{}^{-}(v))\\ &= (\mu_{A_1} \otimes \mu_{A_2})^{-}(u,v) \vee (\mu_{A_1} \circ \mu_{A_2})^{-}(u_1,v)\end{aligned}$$

Hence (u_1, v) dominates (u, v).

Case (ii): $u \in D_1$ and $v \notin D_2$

Let $v_2 \in D_2$, such that

$$\mu_{B_2}{}^{+}(v,v_2) = \min(\mu_{A_2}{}^{+}(v), \mu_{A_2}{}^{+}(v_2))$$

$$\mu_{B_2}{}^{-}(v,v_2) = \max(\mu_{A_2}{}^{-}(v), \mu_{A_2}{}^{-}(v_2))$$

Now $(u, v_2) \in D_1 \times D_2$,

$$\begin{aligned}\mu_B{}^{+}((u,v)(u,v_2)) &= \min(\mu_{A_1}{}^{+}(u), \mu_{B_2}{}^{+}(v,v_2))\\ &= \min((\mu_{A_1}{}^{+}(u), \mu_{A_2}{}^{+}(v)) \wedge \mu_{A_2}{}^{+}(v_2))\\ &= \mu_{A_1}{}^{+}(u) \wedge \mu_{A_2}{}^{+}(v) \wedge \mu_{A_1}{}^{+}(u) \wedge \mu_{A_2}{}^{+}(v_2))\\ &= (\mu_{A_1} \otimes \mu_{A_2})^{+}(u,v) \wedge (\mu_{A_1} \otimes \mu_{A_2})^{+}(u,v_2)\end{aligned}$$

$$\begin{aligned}\mu_B{}^{-}((u,v)(u,v_2)) &= \max(\mu_{A_1}{}^{-}(u), \mu_{B_2}{}^{-}(v,v_2))\\ &= \mu_{A_1}{}^{-}(u) \vee \mu_{A_2}{}^{-}(v)) \vee \mu_{A_2}{}^{-}(v_2))\\ &= \mu_{A_1}{}^{-}(u) \vee \mu_{A_2}{}^{-}(v) \vee \mu_{A_1}{}^{-}(u) \vee \mu_{A_2}{}^{-}(v_2))\\ &= (\mu_{A_1} \otimes \mu_{A_2})^{-}(u,v) \vee (\mu_{A_1} \otimes \mu_{A_2})^{-}(u,v_2)\end{aligned}$$

Hence (u, v_1) dominates (u, v).

Case (iii): $u \notin D_1$ and $v \notin D_2$

D_1 and D_2 be the minimum dominating sets of G_1 *and* G_2.

Therefore, there exist $u_1 \in D_1$ and $v_2 \in D_2$ such that

$$\mu_{B_1}{}^{+}(u\ ,u_1) = \min(\mu_{A_1}{}^{+}(u\ ), \mu_{A_1}{}^{+}(u_1))$$

$$\mu_{B_1}{}^{-}(u\ ,u_1) = \max(\mu_{A_1}{}^{-}(u), \mu_{A_1}{}^{-}(u_1))$$

and $$\mu_{B_2}{}^{+}(v,v_2) = \min(\mu_{A_2}{}^{+}(v), \mu_{A_2}{}^{+}(v_2))$$

$$\mu_{B_2}^{-}(v,v_2) = \max(\mu_{A_2}^{-}(v),\ \mu_{A_2}^{-}(v_2))$$

Let $(u_1, v_2) \in D_1 \times D_2$,

$$\begin{aligned}\mu_B^{+}((u,v)(u_1,v_2)) &= \min(\mu_{B_2}^{+}(u,u_1), \mu_{B_2}^{+}(v,v_{21}))\\ &= \min((\mu_{A_1}^{+}(u) \wedge \mu_{A_1}^{+}(u_1)), (\mu_{A_2}^{+}(v) \wedge \mu_{A_2}^{+}(v_2))\\ &= \mu_{A_1}^{+}(u) \wedge \mu_{A_2}^{+}(v) \wedge \mu_{A_1}^{+}(u) \wedge \mu_{A_2}^{+}(v_2))\\ &= (\mu_{A_1} \otimes \mu_{A_2})^{+}(u,v) \wedge (\mu_{A_1} \otimes \mu_{A_2})^{+}(u_1,v_2)\\ &= \min(\mu_A^{+}(u,v), \mu_A^{+}(u_1,v_2)\end{aligned}$$

and

$$\begin{aligned}\mu_B^{-}((u,v)(u_1,v_2)) &= \max(\mu_{B_2}^{-}(u,u_1), \mu_{B_2}^{-}(v,v_{21}))\\ &= \max((\mu_{A_1}^{-}(u) \vee \mu_{A_1}^{-}(u_1)), (\mu_{A_2}^{-}(v) \vee \mu_{A_2}^{-}(v_2))\\ &= \mu_{A_1}^{-}(u) \vee \mu_{A_2}^{-}(v) \vee \mu_{A_1}^{-}(u) \vee \mu_{A_2}^{-}(v_2))\\ &= (\mu_{A_1} \otimes \mu_{A_2})^{-}(u,v) \vee (\mu_{A_1} \otimes \mu_{A_2})^{-}(u_1,v_2)\\ &= \max(\mu_A^{-}(u,v), \mu_A^{-}(u_1,v_2)\end{aligned}$$

Therefore we get (u_1, v_2) dominates (u,v) in $G_1 \otimes G_2$.

This implies that $D_1 \times D_2$ is a dominating set of $G_1 \otimes G_2$.

Now we prove $D_1 \times D_2$ is minimum.

Let $(x_1, x_2) \in D_1 \times D_2$, $x_1 \in D_1$ and $x_2 \in D_2$.

By our assumption D_1 and D_2 are minimal dominating set of G_1 and G_2 respectively. Therefore $D_1 - x_1$ and $D_2 - x_2$ are not a dominating set. Clearly we get $(D_1 \times D_2) - (x_1 \times x_2)$ is not a minimal dominating set. This implies that $D_1 \times D_2$ is minimal dominating set of $G_1 \otimes G_2$

Therefore $\gamma(G_1 \otimes G_2) = |D_1 \times D_2|$..

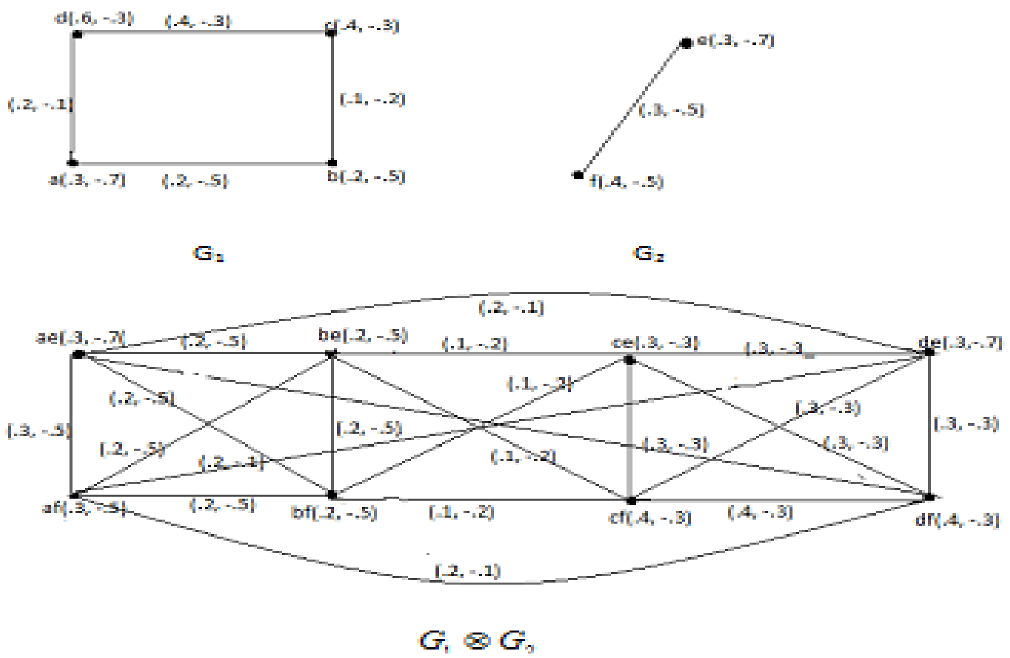


Figure 3.3

In figure 3.3 the minimum dominating set of G_1 and G_2 are D_1={a,c} and D_2={e} respectively. The minimum dominating set of $G_1 \circ G_2$ is {ae,ce}.

References:

1. Akram M. Bipolar fuzzy graphs Information Sci., 2011, 181, 5548–5564.
2. Akram M. Bipolar fuzzy graphs with applications. Knowledge Based Systems, 2013, 39, 1–8.
3. Akram M., Dudek W.A. Interval-valued fuzzy graphs. Comput. Math. Appl., 2011, 61,289–299.
4. Akram M., Dudek W.A. Regular bipolar fuzzy graphs. Neural Computing Appl., 2012, 21,197–205.

5. Akram M., Dudek W.A., Sarwar S. Properties of bipolar fuzzy hypergraphs. Italian J.Pure Appl. Math., 2013, 31, 426–458.
6. Akram M., Li S., Shum K. P. Antipodal bipolar fuzzy graphs. Italian J. Pure Appl. Math., 2013, 31, 425–438.
7. Atanasson , intuitionistic fuzzy set theory and applications, Physcia- verlag, New York, (199).
8. Ayyaswamy.S, and Natarajan.C, Strong (weak) domination in fuzzy graphs, International Journal of Computational and Mathematical sciences, 2010.
9. Balakrishnan and K.Ranganathan, A Text Book of Graph theory, Springer, 2000.
10. Harary.F., Graph Theory, Addition Wesely, Third Printing, October 1972.
11. T.Haynes, S.T.Hedetniemi., P.J.Slater, Fundamentals of Domination in Graph, Marcel Deckker,New York,1998.
12. Rosenfeld A. Fuzzy Graphs ,Fuzzy sets and their Applications (Acadamic Press, New York)
13. Mordeson, J.N., and Nair, P.S., Fuzzy graphs and Fuzzy Hyper graphs, Physica-Verlag, Heidelberg, 1998, second edition, 2001.
14. Nagoor Gani.A and Chandrasekar, Dominations in Fuzzy graph, Advance in Fuzzy sets and systems, 1(1)(2006), 17-26.
15. R.Parvathi and G.Thamizhendhi, Domination in Intuitionistic Fuzzy Graphs, Fourteenth Int. Conf. On IFSs, Sofia 15-16 may 2010.
16. Somasundaram, A., Somasundaram, S., 1998, Domination in Fuzzy Graphs-I, Pattern Recognition Letters, 19, pp. 787–791.
17. Somasundaram, A., 2004, Domination in product Fuzzy Graph-II, Journal of Fuzzy Mathematics
18. R.Parvathi and G.Thamizhendhi, Domination in Intuitionistic Fuzzy Graphs, Fourteenth Int. Conf. On IFSs, Sofia 15-16 may 2010.