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FUZZY MATHEMATICAL MODEL FOR THE EFFECT OF LEPTIN IN BODY WEIGHT MAINTENANCE

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Abstract:

The Present study aims to focus its attention to determine whether Leptin induces Leptin resistance by testing the temporal attenuation of the anorexic and energy expenditure responses to Leptin. Using the logistic distribution, the fuzzy Survival and fuzzy Hazard rate functions were formulated and it is applied to the body mass in the given time interval. The result concludes that when the hazard rate decreases the survival of body weight gain increases in the lower alpha values and vice versa.

Key Words: Survival Function, Hazard Rate Function, Leptin & Logistic Distribution

1. Introduction:

Leptin plays a major role in energy homeostasis. Leptin is a hormone released from fat cells in adipose tissue. Leptin signals to the brain, in particular to an area called the hypothalamus. Leptin does not affect food intake from meal to meal but, instead, acts to alter food intake and control energy expenditure over the long term. Present study has focused on two aspects. Firstly, to investigate the Leptin-induced Leptin resistance, it was examined by the temporal attenuation of the anorexic and energy expenditure responses to Lleptin in mildly obese rats. Secondly, the relative contributions of the anorexic and thermo genic responses to weight loss and weight maintenance over an extended period were analyzed. In particular, we determined whether the thermo genic response alone was sufficient to maintain the reduced weight over an extended period.

2. Fuzzy Mathematical Model:

2.1 Hazard Rate Function:

The survival function examines the chance that breakdowns of organisms, of technical units etc. occur beyond a given point in time. To monitor the lifetime of a unit across the support of its lifetime distribution, the hazard rate h(x) is used. In fact, the hazard rate usually is more informative about the underlying mechanism of failure than the other representatives of a lifetime distribution. For this reason, consideration of the hazard rate may be the dominant method for summarizing survival data. The hazard rate is perhaps the most popular of the six representatives modeling and analyzing lifetime data. This is due to its intuitive interpretation as the amount of risk to fail associated with a unit at age x. Another reason for its popularity is that it is a special case of the intensity function for a non–homogeneous Poisson process. A hazard rate function models the occurrence of only one, namely the first event (= failure), whereas the intensity function models the occurrence of a sequence of events over time.

2.2 Survival Function:

The second lifetime distribution representative is the failure function or lifetime function CDF, defined as

$$F(x) := \Pr(X \leq x),\ x \geq 0, \text{ ------------------------------(1)}$$

Giving the probability of failing up to age x or of having a life span of at most length x.
Another lifetime distribution representative is CCDF, the survival function or reliability function, defined as

$$S(x) := \Pr(X > x),\ x \geq 0, \text{ ------------------------------(2)}$$

Indicating the probability of surviving an age of x or becoming older than x. By examining (1) and (2), we see that the lifetime distribution and the survival function are complementary functions:

$$S(x) = 1 - F(x) \text{ and } F(x) = 1 - S(x) \text{ -------------------(3)}$$

Thus, S(x) is the probability of exceeding x and F(x) is the probability of reaching x. S(x) gives the probability of its functioning at time x and F(x) is the probability of its being down at time x.

2.3 Fuzzy Model:

Fuzzy random variables generalize random variables and random sets. Kwakernaak [6] introduced the concept of a fuzzy random variable as a function: $\Omega \rightarrow (R)$where (Ω, A, P) is a probability triple and $F(R)$ denotes the set of all canonical fuzzy numbers. Fuzzy random variables are only capable of dealing with the vague data. In survival or reliability studies, the hazard rate is an important characteristic. In life testing situations, the expected additional life time given that a component has survived until t is function of t known as mean residual life. The hazard rate and mean residual life have been employed [6] in life length studies and is given by

$$S(x) = \mu/(\mu + \lambda)$$

$$H(x)= 1/(\mu + \lambda)$$

A random variable X follows fuzzy logistic distribution(FLD) with fuzzy parameter($\bar{\bar{\lambda}}$, $\bar{\bar{\mu}}$) is symbolized by $X\sim FLD(\bar{\bar{\lambda}},\ \bar{\bar{\mu}})$.

The fuzzy survival function is given by $\bar{\bar{S}}(x) = \frac{\bar{\bar{\mu}}}{\bar{\bar{\mu}}+\bar{\bar{\lambda}}}$

The alpha cut for the fuzzy survival function is given by $\bar{\bar{S}}(\alpha) = [S_L(\alpha), S_U(\alpha)]$

Where $S_L(\alpha) = \inf\{S(x)|\ \mu\epsilon\bar{\bar{\mu}}(\alpha),\ \lambda\epsilon\bar{\bar{\lambda}}(\alpha)\}$, $S_U(\alpha) = \text{Sup}\{S(x)|\ \mu\epsilon\bar{\bar{\mu}}(\alpha),\ \lambda\epsilon\bar{\bar{\lambda}}(\alpha)\}$

The fuzzy hazard function is given by $\bar{\bar{H}}(x) = \frac{1}{\bar{\bar{\mu}}+\bar{\bar{\lambda}}}$

The alpha cut for the fuzzy hazard function is given by $\bar{\bar{H}}(\alpha) = [H_L(\alpha), H_U(\alpha)]$

Where $H_L(\alpha) = \inf\{H(x)|\ \mu\epsilon\bar{\bar{\mu}}(\alpha),\ \lambda\epsilon\bar{\bar{\lambda}}(\alpha)\}$, $H_U(\alpha) = \text{Sup}\{H(x)|\ \mu\epsilon\bar{\bar{\mu}}(\alpha),\ \lambda\epsilon\bar{\bar{\lambda}}(\alpha)\}$

3. Application:

Leptin resistance is apparent in obese rodents and humans. Rodents made obese by a high-fat diet have impaired responses to exogenously administered leptin, and this leptin resistance becomes more pronounced with progressive degrees of obesity [1],[8]. This form of obesity is accompanied by elevated serum leptin, and that obesity persists despite the elevated leptin, which should promote weight loss [3], [5]. With recombinant adenoviral-mediated leptin gene delivery to lean rats, the suppression in food intake persisted for the 4-wk duration of the experiment [2]. Similarly, with recombinant adeno-associated viral mediated leptin (rAAV-leptin) gene delivery to lean rats, the decrease in food intake persisted for the 6-wk examination period in one study [4]and over a 7.5-wk period in another study [7].

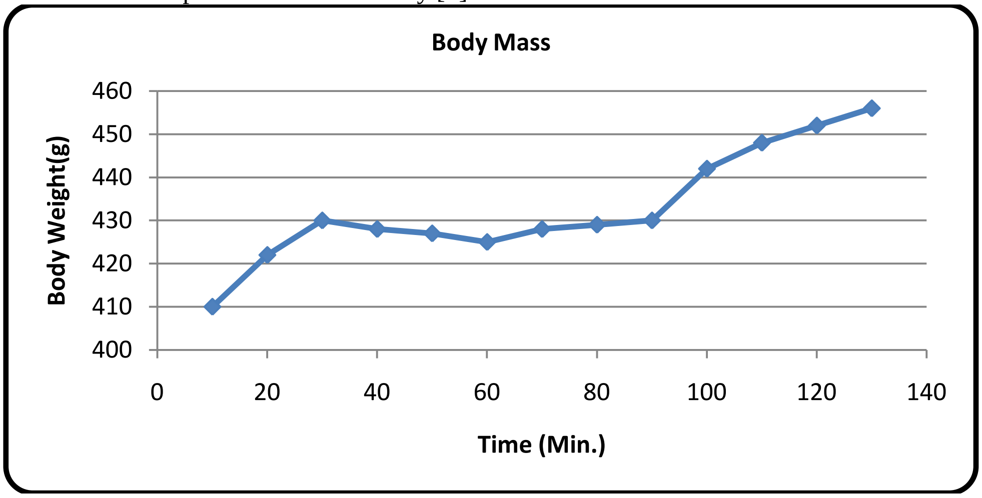

Figure 3.1: The changes of body mass after the administration of leptin

From Fig.3.1, we have the scale and location parameter of logistic distribution

$$\lambda = 0.016 \text{ and } \mu = 6.07$$

Assume that the corresponding triangular fuzzy numbers for the scale and shape parameters are

$$\bar{\lambda} = (0.014, 0.016, 0.018)$$

$$\bar{\mu} = (4.07, 6.07.8.07)$$

and the corresponding α cuts are

$$\bar{\lambda}[\alpha] = [0.014 + 0.002\alpha\,, 0.018 - 0.002\alpha]$$

$$\bar{\mu}[\alpha] = [4.070 + 2\alpha\,, 8.070 - 2\alpha]$$

Table 3.2: Survival and Hazard functions for Lower and Upper α – cut

α	$S_L(\alpha)$	$S_U(\alpha)$	$H_L(\alpha)$	$H_U(\alpha)$
0	0.99657	0.99777	0.24486	0.12364
0.1	0.99673	0.99772	0.23343	0.12677
0.2	0.99688	0.99766	0.22302	0.13007
0.3	0.99701	0.99760	0.21349	0.13355
0.4	0.99713	0.99753	0.20475	0.13721
0.5	0.99725	0.99746	0.19670	0.14108
0.6	0.99735	0.99739	0.18925	0.14518
0.7	0.99745	0.99731	0.18235	0.14952
0.8	0.99754	0.99723	0.17593	0.15413
0.9	0.99762	0.99714	0.16995	0.15903
1	0.99770	0.99704	0.16437	0.16426

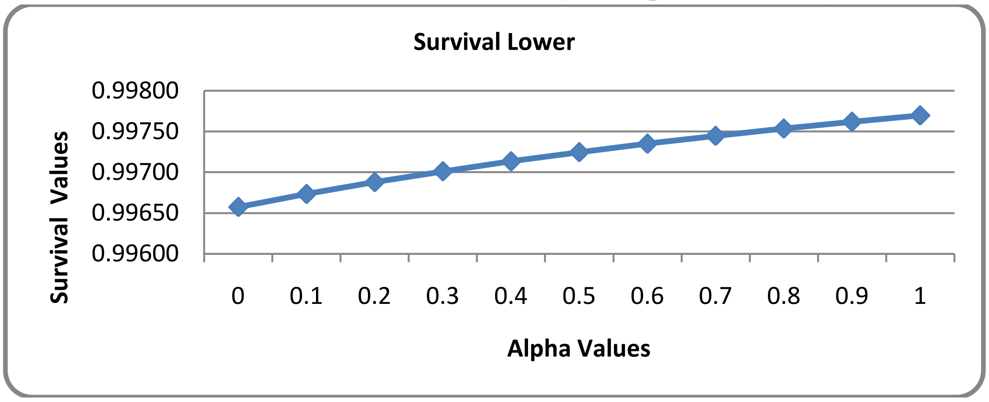

Figure 3.2: The survival rate for lower α – cut values

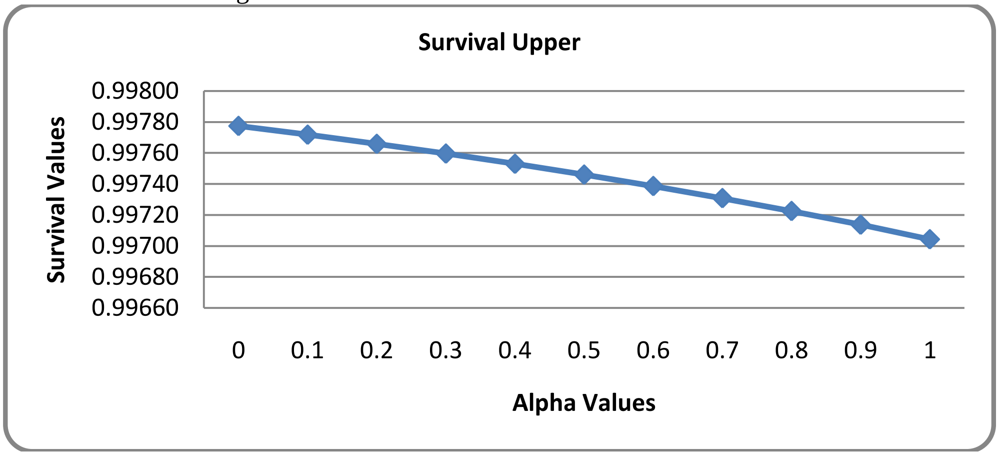

Figure 3.3: The survival rate for upper α – cut values

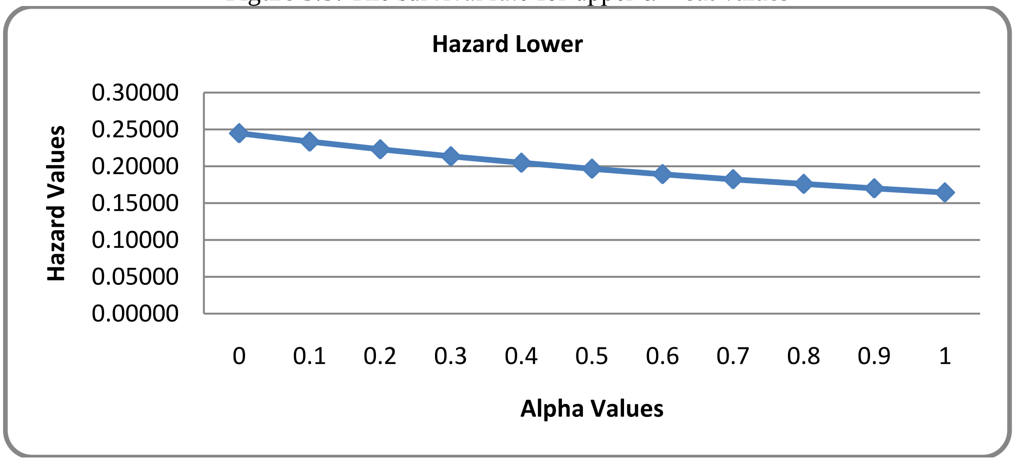

Figure 3.4: The hazard rate for lower α – cut values

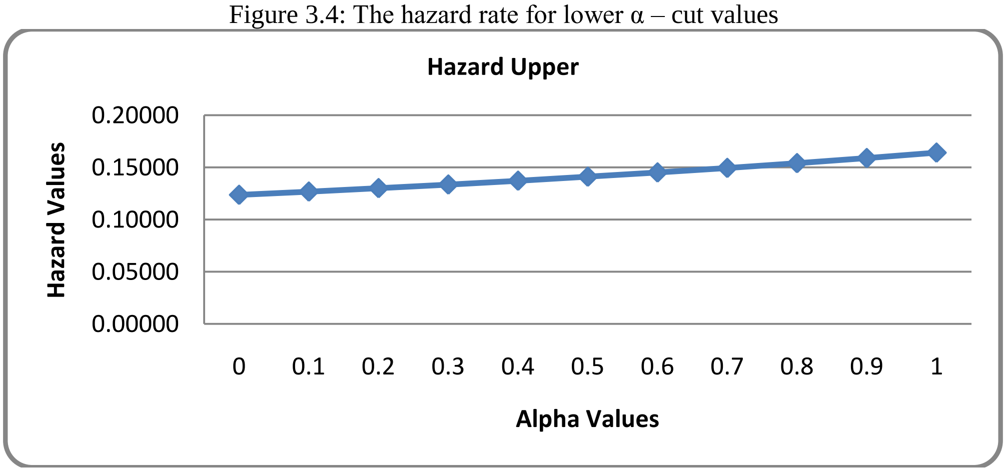

Figure 3.5: The hazard rate for upper α – cut values

4. Conclusion:

The Present study is on fuzzy survival and hazard rate function for the effect of leptin in body weight maintenance. Using the logistic distribution, the Survival and Hazard rate functions of the body mass in the given time intervals were calculated. The result concludes that when the hazard rate decreases the survival of body weight gain increases in the lower alpha values and viceversa.

5. References:

1. Caro JF, Kolaczynski JW, Nycem MR, Ohannesian JP, Opentanova I, Goldman WH, Lynn RB, Zhang PL, Sinha MK, Considine RV 1996 Decreased cerebrospinal-fluid/serum leptin ratio in obesity: a possible mechanism for leptin resistance. Lancet 348:159–161
2. Chen G, Koyama K, Yuanm X, Lee Y, Zhou YT, O'Doherty R, Newgard CB, Unger RH 1996 Disappearance of body fat in normal rats induced by adenovirus-mediated leptin gene therapy. Proc Natl Acad Sci USA 93:14795–14799
3. Considine RV, Sinha MK, Heiman ML, Kriauciunas A, Stephens TW, Nyce MR, Ohannesian JP, Marco CC, McKee LJ, Bauer TL, Carol JF 1996 Serum immunoreactive-leptin concentrations in normal-weight and obese humans. N Engl J Med 334:292–295
4. Dhillon H, Kalra SP, Kalra PS 2001 Dose-dependent effects of central leptin gene therapy on genes that regulate body weight and appetite in the hypothalamus. Mol Ther 2:139–145.
5. Frederich RC, Hamann A, Anderson S, Lollmann B, Lowell BB, Flier JS 1995 Leptin levels reflect body lipid content in mice: evidence for diet-induced resistance to leptin action. Nat Med 1:1311–1314
6. H. Kwakernaak, "Fuzzy random variables. I. Definitions and theorems," Information Sciences, vol. 15, no. 1, pp. 1–29, 1978.
7. Scarpace PJ, Matheny M, Zhang Y, Tümer N, Frase CD, Shek EW, Hong B, Prima V, Zolotukhin S 2002 Central leptin gene delivery evokes persistent leptin signal transduction in young and aged-obese rats but physiological responses become attenuated over time in aged-obese rats. Neuropharmacology 42:549–562
8. Widdowson PS, Upton R, Buckingham R, Arch J, Williams G 1997 Inhibition of food response to intracerebro ventricular injection of leptin is attenuated in rats with diet-induced obesity. Diabetes 46:1782–1785