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INTUITIONISTIC M-FUZZY IDEALS OF SEMIGROUPS

Dr. M. Mary Jansi Rani* & P. Sudhalakshmi**
* Head & Assistant Professor, PG & Research Department of Mathematics, Thanthai Hans Roever College (Autonomous), Perambalur, Tamilnadu
** Research Scholar, PG & Research Department of Mathematics, Thanthai Hans Roever College (Autonomous), Perambalur, Tamilnadu

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Abstract:

In this paper we introduce the concept of Intuitionistic M-fuzzy ideal of semi groups with the concept of several ideal in a semi group and investigate some results.

Key Words: Fuzzy Set, Intuitionistic Fuzzy Set, Intuitionistic Fuzzy Ideal, Intuitionistic Fuzzy (1,2)-Ideal & Intuitionistic Fuzzy Bi-Ideal

Introduction:

After the introduction of the concept of fuzzy sets by Zadeh, several researches were conducted on the generalizations of the notion of fuzzy sets. The concept of "Intuitionistic fuzzy set" was first published by Atanassov [1, 2], as a generalization of the notion of fuzzy sets. Rosenfeld [6] introduced the concept of a fuzzy group and Wu[7] studied the fuzzy normal subgroup. In [4], Kuroki gave some properties of fuzzy ideals and fuzzy bi-ideals in semigroups. In this paper, we introduce the concept of Intuitionistic M-fuzzy ideal of semigroups with the concept of several ideal in a semigroup and investigate some results.

2. Preliminaries:

Definition 2.1: A regular semi group is a semigroup of S in which every element is regular, that is for each element a there exist an element x such that axa=a.

Definition 2.2: An element 'a' of a semigroup S is said to be completely regular if a=axa and ax=xa for some x∈S.

Definition 2.3: An element 'a' of a semi group S is said to be intra regular if a=xa y for some x,y∈S.

Definition 2.4: Let S be any semi group. Then a non-empty set A is said to generate S (or) S is said to be generated by A if every element of S is a finite product of elements of A.

Definition 2.5: A non-empty subset A of a semi group S is said to be a sub semi group of S if a,b∈A => ab∈A

Definition 2.6: A non-empty subset A of a semi group S is said to be a left ideal if SA⊆ A.

Definition 2.7: A non-empty subset A of a semi group S is said to be a right ideal if AS ⊆ A.

Definition 2.8: Let X be a non-empty set. A fuzzy set A on X is a mapping A: X→[0,1] and is defined as A={x∈X / (x,μ(x))}

Definition 2.9: Let G be a group. A mapping μ:G→[0,1] is a fuzzy group if (F G1) μ(xy) ≥min { μ(x) , μ(y) }
(F G2) μ(x) = μ(x) ∀x,y∈G.

3. Intuitionistic Fuzzy Ideals:

Definition 3.1: An IFS A = (μ_A, γ_A) in S is called an intuitionistic fuzzy sub- semi group of S. If
(i)μ_A (xy) ≥min { μ_A(x) , μ_A (y) } and
$(ii)\gamma_A$ (xy) ≤max {γ_A (x) , γ_A (y) } ∀ x,y∈S.

Definition 3.2: An IFS A =(μ_A, γ_A) in S is called an intuitionistic fuzzy left ideal of S if μ_A (xy) ≥μ_A (y)
γ_A(xy) ≤ γ_A(y) for all x,y∈S

An intuitionistic fuzzy right ideal of S is defined in an analogous way.

An IFS A = (μ_A, γ_A) in S is called an intuitisonistic fuzzy ideal of S if it is both an intuitionistic fuzzy right and an intuitionistic fuzzy left ideal of S.

It is clear that any intuitionistic fuzzy left (right) ideal of S is an intuitionistic fuzzy sub semigroup of S.

Definition 3.3: An intuitionistic fuzzy subsemigroup A = (μ_A, γ_A) of S is called an intuitionistic fuzzy bi-ideal of S if
(i) μ_A (xwy) ≥min { μ_A (x) , μ_A (y) } and
(ii) γ_A (xwy) ≤max { (γ_A(x), γ_A(y) } for all w,x,y∈S.

Theorem 3.4:

If S is a regular semigroup then every intuitionistic fuzzy (1, 2) – ideal of S is an intuitionistic fuzzy bi-ideal of S.

Proof:

Assume that a semigroup S is regular and let A = (μ_A, γ_A) be an intuitionistic fuzzy (1,2) – ideal of S. Let w,x,y ∈S. Since S is regular, we have xw ∈(xSx)S xSx, which implies that xw=xSx for sme s∈S.

Thus μ_A (xwy) = μ_A ((xSx)y)
= μ_A (xS(xy))
≥min {μ_A(x), μ_A(x), μ_A(y)}

$= \min \{ \mu_A (x) , \mu_A (y)\}$

And $\gamma_A(xwy) = \gamma_A((xSx)y)$
$= \gamma_A(xS(xy))$
$\leq \max \{\gamma_A (x), \gamma_A (x), \gamma_A (y)\}$
$= \max \{\gamma_A (x), \gamma_A (y)\}$

Therefore A = (μA,γA) be an intuitionistic fuzzy bi-ideal of S.

Hence the Proof

Theorem 3.5:

Every intuitionistic fuzzy bi-ideal of a group S is constant.

Proof:

Let A = (μ_A, γ_A) be an intuitionistic fuzzy bi-ideal of a group S and let x be any element of S.

Then $\mu_A (x) = \mu_A (exe)$
$\geq \min \{ \mu_A (e), \mu_A (e)\}$
$= \mu_A (e)$
$= \mu_A(ee)$
$= \mu_A (xx^{-1}) (x^{-1}x)$
$= \mu_A (x(x^{-1}x^{-1})x)$
$\geq \min \{ \mu_A (x), \mu_A (x)\}$
$= \mu_A(x).$

And $\gamma_A (x) = \gamma_A(exe)$
$\leq \max \{\gamma_A (e), \gamma_A (e)\}$
$= \gamma_A (e)$
$= \gamma_A (ee)$
$= \gamma_A (xx^{-1})(x^{-1}x)$
$= \gamma_A (x(x^{-1}x^{-1})x)$
$\leq \max \{\gamma_A (x), \gamma_A (x)\}$
$= \gamma_A (x)$

Where e is the identity of S.

It follows that $\mu_A (x) = \mu_A (e)$ and
$\gamma_A (x) = \gamma_A (e)$ which means that A $=(\mu_A, \gamma_A)$ is constant.

Hence the theorem.

4. Intuitionistic M-Fuzzy Ideals of Semigroup:

Definition 4.1: An IMFS A $=(\mu_A, \gamma_A)$ in S is called an intuitionistic M fuzzy sub-semigroup of S if
(i) $\mu_A (m(xy) \geq \min \{\mu_A (mx), \mu_A (my)\}$ and
(ii) $\gamma_A (m(xy) \leq \max \{ \gamma_A (mx), \gamma_A (my)\}$ for all x,y∈S.

Definition 4.2: An intuitionistic M- fuzzy sub semigroup A=(μA,γA) of S is called an intuitionistic M fuzzy bi-ideal of S if (i) $\mu_A (m(xwy) \geq \min \{ \mu_A (mx), \mu_A (my)\}$ and
(ii) $\gamma_A (m(xwy) \leq \max \{\gamma_A (mx), \gamma_A (my)\}$ for all w,x,y∈S

Theorem 4.3:

Every intuitionistic M- fuzzy left (right) ideal of S is an intuitioinstic M-fuzzy bi-ideal of S.

Proof:

Let A=(μ_A, γ_A) be an intuitionistic M-fuzzy left ideal of S and w,x,y∈S.

Then $\mu_A (m(xwy)) = \mu_A (m(xw)y)$
$\geq \mu_A (my)$
$\geq \min \{ \mu_A (mx), \mu_A (my)\}$

$\gamma_A (m(xwy)) = \gamma_A (m(xw)y)$
$\leq \gamma_A (my)$
$\leq \max \{\gamma_A (mx), \gamma_A (my)\}$

Thus A = (μ_A, γ_A) is an intuitionistic M-fuzzy bi-ideal of S.

The right case is proved in an analogous way.

$\mu_A (m(xwy) = \mu_A (m(x(wy)))$
$\geq \mu_A (mx)$
$\geq \min \{ \mu_A (mx), \mu_A (my)\}$

$\gamma_A (m(xwy)) = \gamma_A (m(x(wy)))$
$\leq \gamma_A (mx)$
$\leq \max \{ \gamma_A (mx), \gamma_A (my)\}$

Thus A = (μ_A, γ_A) is an intuitionistic M-fuzzy bi-ideal of S.

Hence the theorem.

Theorem 4.4:

Every intuitionistic M- fuzzy bi-ideal is an intuitionistic M- fuzzy (1,2) ideal.

Proof:

Let A = (μ_A, γ_A) be an intuitionistic M- fuzzy bi-ideal of S and let w,x,y,z$\in$ S.

Then μ_A (m(xw(yz))) = μ_A (m(xwy)z)

$\geq$min { μ_A m(xwy) , μ_A (mz)}

$\geq$min {max {μ_A (mx), μ_A (my)}, μ_A (mz)}

=min {μ_A (mx), μ_A (my), μ_A (mz)}

μ_A(m(xw(yz))) $\geq$min {μ_A (mx), μ_A (my), μ_A (mz)}

and γ_A (m(xw(yz))) = γ_A (m(xwy)z)

$\leq$max { γ_A m(xwy), γ_A (mz)}

$\leq$max {max{ γ_A m(x), γ_A m(y)}, γ_A (mz)}

=max {γ_A m(x), γ_A m(y), γ_A (mz)}

γ_A (m(xw(yz))) $\leq$max{ γ_A m(x), γ_A m(y), γ_A (mz)}

Hence the theorem.

Theorem 4.5:

Let A = (μ_A, γ_A) be an intuitionistic M- fuzzy bi-ideal of S. If S is (2,2)- regular then A(mx) = A (mx^2) for all x$\in$S

Proof:

Let x$\in$S

Then there exist a$\in$S such that x= $x^2 ax^2$

Hence μ_A (mx) = μ_Am($x^2 ax^2$)

$\geq$min { μ_A (mx^2) , μ_A (mx^2)}

= μ_A (x^2)

$\geq$min { μ_A (mx) , μ_A (mx)}

= μ_A (mx).

And γ_A (mx) = γ_A m($x^2 ax^2$)

$\leq$max { γ_A (mx^2), γ_A (mx^2)}

=γ_A (mx^2)

$\leq$max { γ_A (mx), γ_A (mx)}

=γ_A (mx)

It follows that

μ_A (mx) = μ_A (mx^2) and

γ_A (mx) = γ_A m(x^2)

So that A(mx)=A(mx^2)

Hence the proof.

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