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A STUDY ON COMBINATORICS INDISCRETE MATHEMATICS

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Abstract:

Discrete mathematics is the study of mathematical structures that are fundamentally discrete rather than continuous. In contrast to real numbers that have the property of varying "smoothly", the objects studied in discrete mathematics – such as integers, graphs, and statements in logic – do not vary smoothly in this way, but have distinct, separated values. The set of objects studied in discrete mathematics can be finite or infinite. The term finite mathematics is sometimes applied to parts of the field of discrete mathematics that deals with finite sets, particularly those areas relevant to business. Research in discrete mathematics increased in the latter half of the twentieth century partly due to the development of digital computers which operate in discrete steps and store data in discrete bits. Concepts and notations from discrete mathematics are useful in studying and describing objects and problems in branches of computer science, such as computer algorithms, programming languages, cryptography, automated theorem proving, and software development. Conversely, computer implementations are significant in applying ideas from discrete mathematics to real-world problems, such as in operations research. Computational geometry has been an important part of the computer graphics incorporated into modern video games and computer-aided design tools. Several fields of discrete mathematics, particularly theoretical computer science, graph theory, and combinatorics, are important in addressing the challenging bioinformatics problems associated with understanding the tree of life.

Introduction:

Combinatorics present approaches for solving counting and structuralquestions. It looks at how many ways a selection or arrangement can be chosen with a specific set of properties and determines if a selection or arrangement of objects exists that has a particular set of properties. They also provides basic information on sets, proof techniques, enumeration and graph theory-topics. The next few chapters explore enumerative ideas, including the pigeonhole principle and inclusion /exclusion. The text then covers enumerative functions and the relations between them. It describes generating functions and recurrences. The authors also present introductions to computer algebra and group theory before considering structures of particular interest in combinatory. Graphs, codes, Latin squares and experimental designs.

Basic Definitions:

Definition 1: A well-defined collection of object is called a Set.
Definition 2: The object in a set are called its member or elements.
Definition 3: The different arrangements which can be made out of a given number of things by taking some or all at a time are called a Permutation.
Definition 4: A group or a selection which can be formed by taking some or all of number of objects irrespective of the order of their arrangements is called a Combination.
Definition 5: The set of integers is the set of positive and negative whole numbers with zero. Therefore the set $\{\ldots.. -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\ldots.\}$
Definition 6: If x is real number the function that assigns the largest integer that is less than or equal to x is called the Floor function of x or Simply the floor of x and denoted by [x].
Definition 7: The continued product of first n natural numbers is called Factorialn and is denoted by the symbol n!. (n! = 1, 2, …… (n−1), n)
Definition 8: A sequence is set of numbers which has one to one correspondence with a set of positive integer. It is denoted by $\{a_n\}$.
Definition 9: Counting process can come to an end is said to be Finite otherwise a set is said to be Infinite.
Definition 10: Two finite sets are said to be Equivalent, if they have the same number of distinct elements.
Definition 11: If A and B be two sets given in such a way that every of A is in B. Then A is a subset of B. Therefore, A⊆B also if A⊆B. Then B is a superset of A.
Definition 12: Which is a superset of each one of the sets under consideration. Such a set is known as Universal Set.
Definition 13: Circle is a plane curve formed by the set of all points of a given fixed distance from a fixed point. The fixed point is called the Centre.
Definition 14: A function which, when represented in terms of infinite series, gives some sequence of functions as the coefficients of the series. This is called Generating function.

Definition 15: A sequence in which the ratio of each term to its preceding term remains the same. This is called Geometric progression.

Definition 16: Probability is a way of expressing knowledge or beliefthatan event will occur or has occurred.

Definition 17: A function of the form $a_0x^n + a_1x^{n-1} + \ldots\ldots + a_n$ where n is positive integer anda_0, a_1, …… a_n are real constants and $a_0 \neq 0$ is called a Polynomialinx of degree n.

Definition 18: When set A and B are disjoint, they have no elements in common.

Definition 19: A non empty set G together with a binary operation *: G×G → G is called a Group. If the following conditions are satisfied.

1. is closed, ab ∈ G for all a, b ∈G
2. is associative, a ∗ (b ∗ c) = (a ∗ b) ∗ c for all a, b, c ∈G
3. There exists an element e ∈ G such that a * e = e * a = a for all a ∈G
4. For any element a in G there exists an element a' ∈G such that a ∗ a' = a'∗ a = e.a' is called the inverse of a.

Types of Permutations Combinations Permutations and Combinations:

Definition 20: An ordered arrangement of r elements of a set containing n distinct elements is called an r-Permutation of n elements and is denoted by P(n, r) or ^nP_r, where r ≤n. An unordered selection of r elements of a set containing n distinct elements is called an r-combination of n elements and is denoted by C(n, r) or n_{C_r} or $\binom{n}{r}$.

Values of P(n, r) and C (n, r):

The first element of the permutation can be selected from a set having n elements in n ways. Having selected the first elements for the first position of the permutation, the second elements can be selected in (n−1) ways, as there are (n−1) elements left in the set. Similarly, there are (n−2) ways of selecting the third element and so on. Finally there are n − (r−1) = n−r +1 ways of selecting the r^{th} element. Consequently, by the product rule, there are,

$$n(n-1)(n-2) \ldots\ldots (n-r+1)$$

Ways of ordered arrangement of r elements of the given set, Thus, P(n, r) = n(n−1)(n−2) …… (n−r +1)

$$P(n, r) = \frac{n!}{(n-r)!}$$

$$P(n, n) = n!$$

Pascal's Identity:

If n and r are positive integers. Where n ≥r, then$\binom{n}{r-1}+\binom{n}{r} = \binom{n+1}{r}$

Proof:

Let S be a set containing (n+1) elements, one of which is 'a',

Let S' = S – {a}. The number of subset of S containing r elements is $\binom{n+1}{r}$. A subset of S with r elements either contains 'a' together with (r-1) elements of S' or contains r elements of S' which do not include 'a'. The number of subsets of (r-1) elements of S' = $\binom{n}{r-1}$There for the number of subsets of r elements of S that contain 'a' = $\binom{n}{r-1}$. Also the number of subset of r elements of S that do no contain 'a' = that of S' = $\binom{n}{r}$.

$$\text{Consequently, } \binom{n}{r-1}+\binom{n}{r} = \binom{n+1}{r}$$

Result: This result can also be proved by using the values of$\binom{n}{r-1}$, $\binom{n}{r}$ and $\binom{n+1}{r}$.

Corollary:

$$C(n+1, r+1) = \sum_{i=r}^{n} C(i, r)$$

Proof:

By Pascal's Identity we get, $\binom{n}{r-1}+\binom{n}{r} = \binom{n+1}{r}$

$$C(n, r-1)+ C(n, r) = C(n+1, r)$$

Changing n to i and r to r+1, we get C(i, r) +C(i, r+1) = C(i+1, r+1)

That is C(i, r) = C(i+1, r+1)−C(i, r+1) -------------------- (1)

Putting i=r, r+1, …… n in (1) and adding, we get, $\sum_{i=r}^{n} C(i, r) = C(n+1, r+1) - C(r, r+1)$

= C(n+1, r+1). [Since C(r, r+1)= 0]

Permutation with Repetition:

Theorem 1: When repetition of n elements contained in a set is permitted in r-permutations, then the number of r-permutations is n^r.

Proof: The number of r-permutation of n elements can be considered as the same as the number of ways in which the n elements can be placed in r positions.The first position can be occupied in n ways, as any one of the n elements can be used. Similarly, the second position can also be occupied in n ways, as any one of the n elements can be used, since repetition of elements is allowed. Hence, the first two positions can be occupied in n ×n = n^2 ways, by the product rule. Proceeding like this, the 'r' positions can be occupied by 'n' elements with repetition in n^r ways. The number of r- permutations of n elements with repetition = n^r.

Theorem 2: The number of different permutations of n object which include n_1 identical objects of type I, n_2 identical objects of type II,….. and n_k identical objects of type k is equal to $\frac{n!}{n_1!n_2!.....n_k!}$, where $n_1 + n_2 + \cdots..+n_k=n$.

Proof: The number of n-permutations of n objects is equal to the number of ways in which the n objects can be placed in n positions. n_1Positions to be occupied by n_1 objects of the I type can be selected from n positions in C(n, n_1) ways. n_2Positions to be occupied by the n_2 objects of the II type can be selected from the remaining $(n-n_1)$ positions in $C(n-n_1.n_2)$ ways and so on. Finally n_k positions to be occupied by the n_k objects of type k can be selected from the remaining $(n-n_1-n_2-\ldots\ldots-n_{k-1})$ position in $(n-n_1-n_2-\ldots\ldots-n_{k-1}n_k)$ ways.

Hence, the required number of different permutations

$$= C(n, n_1)\times C(n-n_1,n_2)\times \ldots\ldots C(n-n_1-n_2-\ldots\ldots-n_{k-1,}n_k)$$

Since by the product rule, $C(n, r) = \frac{n!}{r!(n-r)!}$

$$= \frac{n!}{n_1!(n-n_1)!} \times \frac{(n-n_1)!}{n_2!(n-n_1-n_2)!} \times \ldots\ldots\times \frac{(n-n_1-n_2-\cdots\ldots-n_{k-1}n_k)!}{n_k!0!}$$

$(Since n_1+n_2+\ldots..+n_k=n)$

$$= \frac{n!}{n_1!n_2!.....n_k!}$$

Recurrence Relation by Using Generating Functions Statement of the Principle of Mathematical Induction:

Let S(n) denote a mathematical statement or a set of such statements that involves one or more occurrence of the variable n, which represents a positive integer.

- ✓ If S (1) is true and If, whenever S(k) is true for some particular, but arbitrarily chosen k∈Z*, S(k+1) is also true, then S(n) is true for all n∈Z*.

Strong From of the Principle:

Given a mathematical statement S(n) that involves one or more occurrences of the positive integer n and if

- ✓ S(1) is true and Whenever S(1), S(2), ……S(k) are true , S(k+1) is also true, then S(n) is true for all nϵZ*.

Recurrence Relations:

Definition: An equation that expressa$_n$, the general term of the sequence $\{a_n\}$ in terms of one or more of the previous terms of the sequence, namely $a_0, a_1, \ldots . a_{n-1}$ for all integers n with $n\geq n_0$, where n_0 is the non-negative integers is called a recurrence relation for $\{a_n\}$ or a difference equation.

Definition: If the terms of a sequence satisfy a recurrence relation, then the sequence is called a solution of the recurrence relation.

The assumption that f(n) is not a solution of the associated homogeneous relation:

Form of f(n)	**Form of** $a_n^{(p)}$ **to be assumed**
C, a constant	A, a constant
n	$A_0n + A_1$
n^2	$A_0n^2 + A_1$
$n^t, t \in Z*$	$A_0n^t + A_1n^{t-1} + \cdots + A_n$
$r^n, r \in R$	Ar^n
n^tr^n	$r^n(A_0n^t + A_1n^{t-1} + \cdots + A_n)$
Sin α n	A sin α n + B cos α n
Cos α n	A sin α n +cos α n
r^n Sin α n	r^n (A sin α n + B cos α n)
r^nCos α n	r^n(A sin α n +cos α n)

Conclusion:

A study of this project is combinatorics in discrete mathematics. We conclude deals with types of permutations combinations and itsapplication expressed and deals with solutions of recurrence relation byusing generating functions.

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