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INVENTORY MODEL WITH VARIABLE RATE OF DETERIORATION

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Abstract:

Operations Research is the scientific knowledge through interdisciplinary team effort for the purpose of determining the best utilization of limited resources. Nowadays operations research is having more applications in engineering, economics, business management, agriculture etc. Operations research utilizes various techniques like linear programming, integer programming, inventory control, queuing theory game theory, goal programming, non linear programming's. Inventory control deals with determining the level of a commodity that a business must maintain to ensure smooth operation. The basis for the decision is a model from holding too much inventory against the penalty cost resulting from inventory shortage .the principal factor affecting the solution is the nature of the demand which is either deterministic or probabilistic. In real life, demand is usually probabilistic. But in some cases the simpler deterministic approximation may be acceptable. Inventory is defined as any idle resource of enterprise. It is a physical stock of goods kept for future use. In a factory, the inventory may be in the form of raw materials, spare parts, semi-finished goods etc. An inventory may also include furniture, machinery.

Key Words: Inventory, Cost, Demand, EOQ & Minimizing.

Introduction:

Inventory is defined as any idle resource of enterprise. It is a physical stock of goods kept for future use. In a factory, the inventory may be in the form of raw materials, spare parts, semi-finished goods etc. An inventory may also include furniture, machinery etc. The need of the management to make decisions regarding an inventory arises because of the various alternative courses of action available with the enterprise. It is essential for an enterprise to have an inventory, due to the following reasons.

Inventory Model:

Inventory is essential to provide flexibility in operation a system or organization. An inventory can be classified into raw materials inventory, work – in – process inventory and finished goods inventory.

- ✓ The raw – material inventory removes dependency between suppliers and plants.
- ✓ The work – in - process inventory removes dependency between plants and its customers or market.

The Main Functions of an Inventory are:

Smoothing out irregularities in supply, minimizing the production cost and allowing organizations to cope up with perishable materials.

Models of Inventory:

There are different models of inventory. The inventory models can be classified into deterministic models and probability models. The various deterministic models are:

- ✓ Purchase model with instantaneous replenishment and without shortages
- ✓ Manufacturing model without shortage
- ✓ Purchase model with instantaneous replenishment with shortages
- ✓ Manufacturing model with shortages.

Basic Definitions:

Set Up Cost - The cost associated with the setting up of machinery before starting production is termed as set up cost.

Ordering Cost - The cost associated with ordering of raw material for production purposes, advertisements, consumption of stationery and postage, telephone charges, rent for space used by purchasing department etc constitute the ordering cost.

Production Cost - The cost of producing a unit of an item is known as production cost. This cost generally includes the salary paid to the employees, depreciation of equipments, electricity charges, production taxes etc.

Holding Cost - The costs associated with the storage of inventory until its use or sales are termed as holding cost. This cost generally includes the cost such as rent for space used for storage, interest on the money locked-up, insurance of stored equipments, furniture used etc.

Shortage Cost - The penalty cost for running out of stock is known as shortage cost. This cost includes the loss of potential profit through sales of items and loss of goodwill in terms of permanent loss of customers and its associated lost profit in future sales.

Deterioration Cost - Deterioration cost is the loss incurred due to deterioration of the items in the inventory while storage.
Demand - The number of units required per period is called demand.
Static Demand - If the demand is known and is a constant over time then it is said to be static.
Dynamic Demand - If the demand is known and varies with time then it is said to be dynamic.
Ramp Type Demand - Ramp type demand is one in which demand increases up to a certain time after which it stabilizes and become constant.
Lead Time - The time gap between placing of an order and the actual arrival of the inventory is known as lead time.
Order Cycle - The time period between the placement of two successive orders is referred to as an order cycle.
Continuous Review - The record of the inventory level is checked continuously until a certain lower limit is reached before a new order is placed. This is often known as a two-bin system.
Periodic Review - The inventory levels are reviewed at equal time intervals and orders are placed at such intervals.
Time Horizon - The time period over which the inventory level will be controlled is known as time horizon.
Recorder Level - The level between the maximum and the minimum stock, at which the purchasing activities for replenishment must begin, is known as recorder level.
Re order point - The inventory level at which an order should be placed is the reorder point shortage. The unsatisfied quantity of demand due to insufficient inventory in hand is known as shortage.
Discount - The reduction in the unit price for large orders is referred to as discount.

EOQ Model Having Constant Demand:

In this chapter, inventory model has been developed for deteriorating items assuming the demand rate to be constant for some time and then as a linear function of time. The optimal value for the length of the replenishment cycle and the optimum order quantity are determined so that the total inventory cost is minimized.

Notations:

The notations below are used to define the parameters.

$I(t)$ − On hand inventory level at time t
T − Length of the replenishment cycle
q − Number of items at the beginning of the period
μ − The time point at which demand increases with time and also deterioration starts
θ − Deterioration rate
C − Unit cost
C_1 − Inventory holding cost per unit per unit time
C_3 − Set-up cost per cycle
$K(t)$ − The total cost of the system per unit time
T^* − Optimum value of T
q^* − Optimum value of q
$K(t^*)$ − Optimum total cost per unit time

Assumptions:

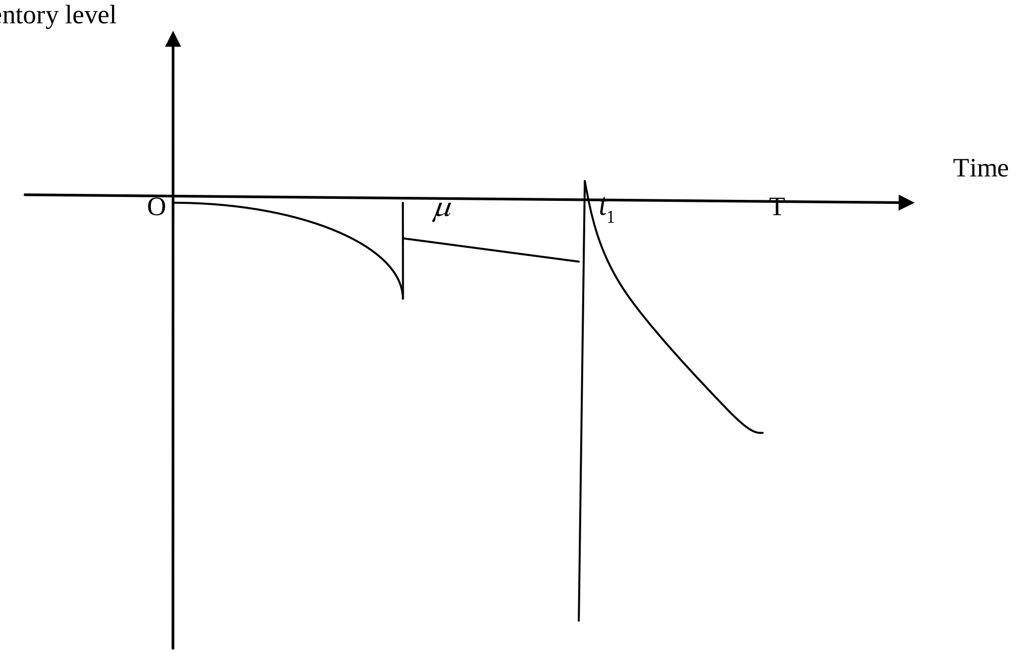
The following are the assumptions made in this chapter.

- ✓ A single item is considered over a prescribed period of T units of time
- ✓ The replenishment occurs instantaneously at an infinite rate
- ✓ Delivery lead time is zero.
- ✓ There is no repair or replacement of deteriorated units.
- ✓ The time horizon of the inventory system is infinite. Only a typical planning schedule of length T is considered, all the remaining cycles are identical.
- ✓ For the time interval $[0,\mu]$, demand is constant at the rate of a units per unit of time. For the time interval $[\mu,T]$, demand rate is a linear function of time, with the form as
$$R(t)=a+b(t-\mu),\mu<t\leq T$$
- ✓ The holding cost, ordering cost and unit cost remain constant over time.

There is no deterioration for the period $[\mu,T]$ which is practically very small.

Development of the Model: The inventory system starts with zero inventory at t=0. Shortages are allowed to accumulate upto timet_1 At time t_1 inventory is procured. The quantity received at t_1 is used partly to meet the shortages which are accumulated in the previous cycle from time 0 to t_1 and the rest of the quantity is used to meet the demand and deterioration in $[t_1, T]$. The inventory level gradually falls to zero at time T.

The inventory system developed is depicted by the following figure.



Inventory Level- Time Relationship:

The instantaneous states of the inventory level I(t) at any time t is given by the following differential equations.

$$\frac{dI(t)}{dt} = -Ae^{bt}, \qquad 0 \le t \le \mu \tag{1}$$

$$\frac{dI(t)}{dt} = -Ae^{b\mu}, \qquad \mu \le t \le t_1 \tag{2}$$

$$\frac{dI(t)}{dt} + \alpha\beta t^{\beta-1} I(t) = -Ae^{b\mu}, \qquad t_1 \le t \le T \tag{3}$$

With boundary conditions given by I(0)=0 and I(T)=0 .

The solution of the differential equation (4.1) is given by

$$I(t) = \frac{A}{b}\left(1 - e^{bt}\right), \; 0 \le t \le \mu \tag{4}$$

Now consider equation (4.2)

$$\frac{dI(t)}{dt} = -Ae^{b\mu}, \qquad \mu \le t \le t_1$$

Then $I(t) = -Ae^{b\mu} t + C$

Using equation (4), $C = \frac{A}{b}\left(1 - e^{b\mu}\right) + A\mu e^{b\mu}$.

Thus
$$I(t) = -Ae^{b\mu} t + \frac{A}{b}\left(1 - e^{b\mu}\right) + A\mu e^{b\mu} \tag{5}$$
$$= Ae^{b\mu}(\mu - t) + \frac{A}{b}\left(1 - e^{b\mu}\right), \mu \le t \le t_1$$

Similarly the differential equation (3) gives the solution as

$$I(t)e^{\alpha t^{\beta}} = \int -Ae^{b\mu} e^{\alpha t^{\beta}} dt + C$$
$$= -Ae^{b\mu}\left[t + \frac{\alpha t^{\beta+1}}{\beta+1}\right] + C$$

$I(T) = 0$ gives $C = Ae^{b\mu}\left[T + \frac{\alpha}{\beta+1} T^{\beta+1}\right]$.

Therefore, $I(t)e^{\alpha t^{\beta}} = Ae^{b\mu}\left[(T-t)+\frac{\alpha}{\beta+1}\left(T^{\beta+1}-t^{\beta+1}\right)\right]$.

And hence $I(t) = Ae^{b\mu-\alpha t^{\beta}}\left[(T-t)+\frac{\alpha}{\beta+1}\left(T^{\beta+1}-t^{\beta+1}\right)\right], t_1 \le t \le T$ (6)

When $t > t_1 > \mu, H(t-\mu)=1$, Thus $R(t) = Ae^{b\mu}$.

Demand during $[t_1, T] = Ae^{b\mu}(t-t_1)$ (7)

The total number of items deteriorates during $[t_1, T]$ = Inventory at t_1 – Demand during $[t_1, T]$.

Using (6) and (7) the number of items that deteriorate during $[t_1, T]$ is found as

$$\chi_d = Ae^{b\mu-\alpha t_1^{\beta}}\left[(T-t_1)+\frac{\alpha}{\beta+1}\left(T^{\beta+1}-t_1^{\beta+1}\right)\right] - Ae^{b\mu}(T-t_1) \tag{8}$$

The inventory held over the period $[t_1, T]$ is

$$\chi_h = \int_{t_1}^{T} Ae^{b\mu-\alpha t^{\beta}}\left[(T-t)+\frac{\alpha}{\beta+1}\left(T^{\beta+1}-t^{\beta+1}\right)\right]dt$$

$$= \int_{t_1}^{T} ATe^{b\mu}e^{-\alpha t^{\beta}}dt - \int_{t_1}^{T} ATe^{b\mu}e^{-\alpha t^{\beta}}t\,dt + \int_{t_1}^{T} Ae^{b\mu}\frac{\alpha}{\beta+1}T^{\beta+1}e^{-\alpha t^{\beta}}dt - \int_{t_1}^{T} Ae^{b\mu}\frac{\alpha}{\beta+1}t^{\beta+1}e^{-\alpha t^{\beta}}dt$$

Consider

$$\int_{t_1}^{T} ATe^{b\mu}e^{-\alpha t^{\beta}}dt = ATe^{b\mu}\int_{t_1}^{T} e^{-\alpha t^{\beta}}dt$$

Using the power series representation of $e^{-\alpha t^{\beta}}$ and neglecting higher powers of α, it is seen that

$$\int_{t_1}^{T} ATe^{b\mu}e^{-\alpha t^{\beta}}dt$$

$$= ATe^{b\mu}\int_{t_1}^{T}(1-\alpha t^{\beta})dt$$

$$= ATe^{b\mu}\left[t-\frac{\alpha t^{\beta+1}}{\beta+1}\right]_{t_1}^{T}$$

$$= ATe^{b\mu}\left[(T-t_1)-\frac{\alpha}{\beta+1}\left(T^{\beta+1}-t_1^{\beta+1}\right)\right]$$

Consider $\int_{t_1}^{T} Ae^{b\mu}e^{-\alpha t^{\beta}}tdt$

$$= Ae^{b\mu}\int_{t_1}^{T}\left(1-\alpha t^{\beta}\right)tdt$$

$$= Ae^{b\mu}\left[\frac{t^2}{2}-\alpha\frac{t^{\beta+2}}{\beta+2}\right]_{t_1}^{T}$$

$$= Ae^{b\mu}\left[\frac{1}{2}(T^2-t_1^2)-\frac{\alpha}{\beta+2}\left(T^{\beta+2}-t_1^{\beta+2}\right)\right]$$

Consider $\int_{t_1}^{T} Ae^{b\mu}\frac{\alpha}{\beta+1}T^{\beta+1}e^{-\alpha t^{\beta}}dt$

$$= Ae^{b\mu}\frac{\alpha}{\beta+1}T^{\beta+1}\left[t-\frac{\alpha t^{\beta+1}}{\beta+1}\right]_{t_1}^{T}$$

$$= Ae^{b\mu}\frac{\alpha}{\beta+1}T^{\beta+1}\left[(T-t_1)-\frac{\alpha}{\beta+1}\left(T^{\beta+1}-t_1^{\beta+1}\right)\right]$$

Consider $\int_{t_1}^{T} Ae^{b\mu}\frac{\alpha}{\beta+1}t^{\beta+1}e^{-\alpha t^{\beta}}dt$

Conclusion:

In all the chapters we have considered inventory models for deteriorating items. Various types of demand functions such as ramp type demand, exponential demand and time dependent demand etc are taken into consideration. In all the cases depending upon the demand the optimal order quantity is determined so that the total cost is minimum.These results can be implemented in real life situation and are reliable. Using these inventory models decisions regarding how much goods to be produced or raw materials to be ordered can be made judiciously with more economic benefits.The above inventory models can be applicable for food range like paddy, rice, wheat, etc., as the demand of the food grains increases with time for a fixed time horizon. Thus Inventory models provide a quantitative decision making aids and contributes to optimal control of an inventory system. These models when suitably applied can lead to significant improvements and increased profitability.Study of these works create interest to develop inventory models under various constraints for day to day needs in industries and factories. Depending upon the need of real life situation more parameters can be considered and the expression for optimal order quantity can be derived.

References:

1. Ghosh, S. K. and Chaudhuri, K. S. (2004): An order-level inventory model for a deterioration item with Weibull distribution deterioration. Time-quadratic demand and shortages;
2. Aggarwal S. P, “A note on an order-level inventory model for a system with constant rate of deterioration”.
3. ACCST Research Journal [vol. V(2), April]
4. Shah Y.K and Jaiswal M. C “Order –level inventory model for a system with constant rate of deterioration”, Opsearch, 14 (1977).
5. Kun-Shan WU: An EOQ inventory model for items with Weibull distribution deterioration, ramp type demand rate and partial backlogging, production planning and control, 12(2001)
6. Perumal V & Arivarignan G (2002): A production model with two rates of productions and backorders. International Journal of Management system (18).