International Journal of Computational Research and Development (IJCRD)
Impact Factor: 5.015, ISSN (Online): 2456 - 3137
(www.dvpublication.com) Volume 3, Issue 2, 2018

REPRESENTATION AND OPERATION ON FUZZY MATRICES

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Cite This Article: D. Girija & B. Amudha, "Representation and Operation on Fuzzy Matrices", International Journal of Computational Research and Development, Volume 3, Issue 2, Page Number 1-5, 2018.

Abstract:

The concepts of fuzzy matrices and the operations on any two fuzzy matrices are analyses in this chapter. Also we prove that the set of fuzzy matrices forms a lattice under matrix minima and matrix maxima binary fuzzy operations. Fuzzy matrices were introduced for the first time by Thomson, who discussed the convergence of powers of fuzzy matrix. Fuzzy matrices play an important role in scientific development. The basic and essential fuzzy matrix theory is given. Instead, the authors have only tried to give those essential basically needed to develop the fuzzy model. Fuzzy set and fuzzy logic was introduced by Professor Lofti A Zadeh in 1965. The success of research in fuzzy sets and fuzzy logic has been demonstrated in a variety of fields, such as artificial intelligence, computer science, control engineering, computer application and robotics

Key Words: Fuzzy Matrices & Matrix

Introduction:

Fuzzy matrices were introduced for the first time by Thomson, who discussed the convergence of powers of fuzzy matrix. Fuzzy matrices play an important role in scientific development. The basic and essential fuzzy matrix theory is given. Instead, the authors have only tried to give those essential basically needed to develop the fuzzy model. Fuzzy set and fuzzy logic was introduced by Professor Lofti A Zadeh in 1965. The success of research in fuzzy sets and fuzzy logic has been demonstrated in a variety of fields, such as artificial intelligence, computer science, control engineering, computer application, robotics and many more. This paper aims to assist social to analyze their problems using fuzzy models. The basic and essential fuzzy matrix theory is given. The paper does not promise to give the complete properties of basic fuzzy theory or fuzzy matrices. Instead, the authors have only tried to give those essential basically needed to develop, the fuzzy model. The authors do not present elaborate mathematical theories to work with fuzzy matrices. Instead they have given only the needed properties by way of examples. The authors feel that the paper should mainly help social scientists, who are interested in finding out ways to emancipate the society. Everything is kept at simplest level and even difficult definitions, have been omitted. Another main feature of this paper is the description of each fuzzy model using examples from real- world problems. Further, this paper gives lots of reference so that the interested reader can make use of them.

Operations on Fuzzy Matrices:

The concepts of fuzzy matrices and the operations on any two fuzzy matrices are analyses in this chapter. Also we prove that the set of fuzzy matrices forms a lattices under matrix minima and matrix maxima binary fuzzy operations.

Definition:

Two fuzzy matrices x and y are compatible under matrix addition. If they are of same order.

Example:

Let us define two fuzzy matrices x and y.

$$X=\begin{pmatrix} 0.3 & 0.7 & 0.8 & 0.9 \\ 0.4 & 0.5 & 1 & 0.3 \\ 0.6 & 0.1 & 0.4 & 0.8 \\ 0.3 & 0.4 & 0.6 & 0.2 \end{pmatrix}$$

$$Y=\begin{pmatrix} 1 & 0.2 & 0.4 & 0.9 \\ 0.3 & 0.6 & 0.1 & 0.3 \\ 0.8 & 0.9 & 0.5 & 0.6 \\ 0.3 & 0.2 & 0.5 & 0.7 \end{pmatrix}$$

Then the addition of two matrices

$$X+Y=\begin{pmatrix} 1.3 & 0.9 & 1.2 & 1.8 \\ 0.7 & 1.1 & 1.1 & 0.7 \\ 1.4 & 1 & 0.9 & 1.4 \\ 0.6 & 0.6 & 1.1 & 0.9 \end{pmatrix}$$

Clearly X+Y is a matrix, but not a fuzzy matrix. Hence we can conclude that addition of two fuzzy matrices compatible under addition need not be a fuzzy matrix.

Maximum Operation of Two Fuzzy Matrices:

Two fuzzy matrices are conformable for operation if they are of the same order. Hence for two matrices) X= (X_{ij}) and Y= (Y_{ij}) of order M×N, maxima of these two matrices is a max(X, Y) = (C_{ij}) of order M×N, where (C_{ij}) = max (X_{ij},Y_{ij}).Hence for the matrices, Let us define two fuzzy matrices x and y.

$$X=\begin{pmatrix} 0.3 & 0.7 & 0.8 & 0.9 \\ 0.4 & 0.5 & 1 & 0.3 \\ 0.6 & 0.1 & 0.4 & 0.8 \\ 0.3 & 0.4 & 0.6 & 0.2 \end{pmatrix}$$

$$Y=\begin{pmatrix} 1 & 0.2 & 0.4 & 0.9 \\ 0.3 & 0.6 & 0.1 & 0.3 \\ 0.8 & 0.9 & 0.5 & 0.6 \\ 0.3 & 0.2 & 0.5 & 0.7 \end{pmatrix}$$

$$\text{Max } X + Y=\begin{pmatrix} 1 & 0.7 & 0.8 & 0.9 \\ 0.4 & 0.6 & 1 & 0.4 \\ 0.8 & 0.9 & 0.5 & 0.8 \\ 0.9 & 0.4 & 0.6 & 0.7 \end{pmatrix}$$

Maximum Operation of Two Fuzzy Matrices:

Two fuzzy matrices are conformable for minimum operation if they are of same order. Hence for two matrices X= (X_{ij}) and Y= (Y_{ij}) of order M×N, fuzzy minima of these two matrices is a matrix min(X, Y) = (C_{ij}) of order M×N. where, C_{ij}= MIN (X_{ij},Y_{ij}). Hence for the matrices, C_{ij}= MIN (X_{ij},Y_{ij}). Hence for the matrices, Let us define two fuzzy matrices x and y.

$$X=\begin{pmatrix} 0.3 & 0.7 & 0.8 & 0.9 \\ 0.4 & 0.5 & 1 & 0.3 \\ 0.6 & 0.1 & 0.4 & 0.8 \\ 0.3 & 0.4 & 0.6 & 0.2 \end{pmatrix}$$

$$Y=\begin{pmatrix} 1 & 0.2 & 0.4 & 0.9 \\ 0.3 & 0.6 & 0.1 & 0.3 \\ 0.8 & 0.9 & 0.5 & 0.6 \\ 0.3 & 0.2 & 0.5 & 0.7 \end{pmatrix}$$

$$\text{MIN } X + Y=\begin{pmatrix} 0.3 & 0.2 & 0.4 & 0.9 \\ 0.3 & 0.5 & 0.1 & 0.3 \\ 0.6 & 0.1 & 0.4 & 0.6 \\ 0.3 & 0.2 & 0.5 & 0.2 \end{pmatrix}$$

In case of fuzzy matrices, we have seen that the addition is not defined, where as the maxima and minima operations are defined. Clearly under the maximum minimum operations the resultant matrix is again a fuzzy matrix of the same order and max(X,Y) ≠ min(X,Y).

Definition:

Let x = (X_{ij})M×N be a fuzzy matrices, the matrix obtained by replacing each element xij of X by its dual (1−X_{ij}) is called conjugate fuzzy matrix of x and is denoted as X conjugate of matrices x and y satisfies De Morgan's rule under fuzzy matrix product of max=min and min − max operations.

(i) $\overline{(\max - \min (xy)}$ = min − max($\overline{xy}$)

(ii) $\overline{(\min - \max (xy)}$ = max − min($\overline{xy}$)

Example:

D_{11} = max {min (0.3,1),min)(0.7,0.3),min (0.8,0.8),min (0.9,0.3)}
= max {0.3,0.3,0.8,0.3}
= 0.8
= 0.2

D_{12} =max {min (0.3,0.2),min)(0.7,0.6),min (0.8,0.9),min (0.9,0.2)}
= max {0.2,0.6,0.8,0.2}
= 0.8
= 0.2

D_{13} =max {min (0.3,0.4),min)(0.7,0.1),min (0.8,0.5),min (0.9,0.8)}
= max {0.3,0.1,0.5,0.5}
= 0.5
= 0.5

D_{14} =max {min (0.3,0.9),min)(0.7,0.4),min (0.8,0.6),min (0.9,0.7)}
= max {0.3,0.4,0.6,0.7}
= 0.7
= 0.3 and so on.

Hence,

$$\overline{\text{max - min (XY)}} = \begin{pmatrix} 0.2 & 0.2 & 0.5 & 0.3 \\ 0.2 & 0.1 & 0.5 & 0.4 \\ 0.4 & 0.6 & 0.5 & 0.3 \\ 0.1 & 0.4 & 0.5 & 0.1 \end{pmatrix}$$

$$\text{min - max } \overline{(XY)} = \begin{pmatrix} 0.2 & 0.2 & 0.5 & 0.3 \\ 0.2 & 0.1 & 0.5 & 0.4 \\ 0.4 & 0.6 & 0.5 & 0.3 \\ 0.1 & 0.4 & 0.5 & 0.1 \end{pmatrix}$$

Lattices of Fuzzy Matrices:

Definition:

A partial ordered set in which every pair of elements has both least upper bound and greatest lower bound is called a lattice.

(i) Idempotent (A ∧ A =A (or) A ∨ A = A)
(ii) Commutative (A∨ B = B ∨ A (or) A ∧ B = B ∧A)
(iii) Associative (A∨(B∨C) = (A∨ B) ∨ C (or) A ∧(B∧C) = (A∧B)∧ C)
(iv) Absorption law (A∨(A∧B) =A (or) A∧(A∨ B) = A).

Theorem:

Let A= {Ai}I∈J is a set of m×n fuzzy matrices where J= {1,2,…..}, then under matrix minima ∧ and matrix maxima ∨ operations A forms a lattice.

Proof:

To prove that the given set A of fuzzy matrices forms a lattice, we shall show that A satisfies the four properties (i) to (iv).

(i) Idempotent Law:

For any fuzzy matrix Ai ∈ A, the following holds:
Min (Ai, Ai) = Ai and max (Ai, Ai) = Ai.
Hence Idempotent Law is satisfied.

(ii) Commutative Law:

It can be easily verified that for all the fuzzy matrices Ai and Ai ∈ A, the following holds
Min (Ai, Aj) = min (Aj, Ai) and max (Ai, Aj) = max (Aj, Ai).
This proves that Commutative Law is satisfied.

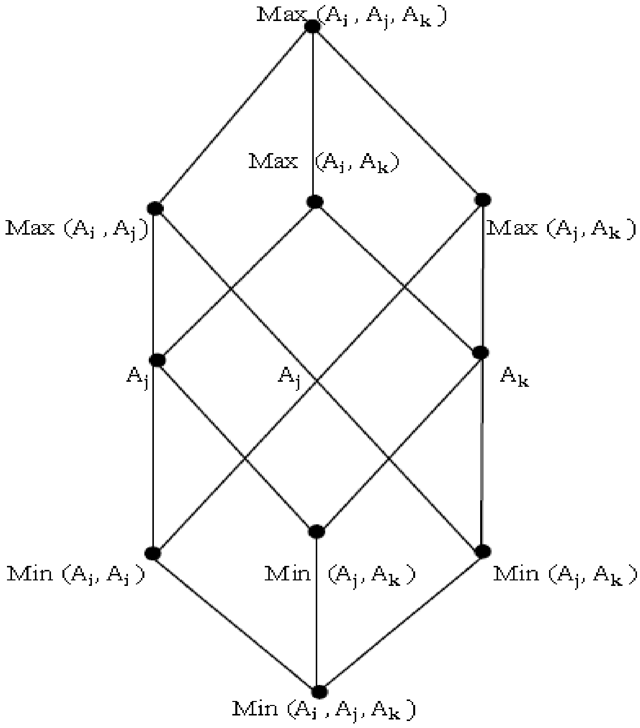
(iii) Associative Law:

For any fuzzy matrices Ai, Aj, Ak∈ A, it can be proved that
Min ((Ai, Aj), Ak) = min (Ai ,(Aj, Ak)) and
Max ((Ai, Aj), Ak) = max (Ai ,(Aj, Ak)) .
Hence we can conclude that A associative under matrix maxima and matrix minima operation.

(iv) Absorption Law:

For any fuzzy matrices Ai, Aj, we can prove that
Min (Ai, max (Ai, Aj)) = Ai and max (Ai, min (Ai, Aj)) = Ai.
Hence Absorption Law holds.

Since it satisfies all the four properties (i) to (iv) of the lattices, therefore the set of the matrices is a lattice under matrix maxima and matrix minima.



A lattice of fuzzy matrices under matrix maxima and matrix minima

Representation of Fuzzy Matrix Based on Reference Function:

Fuzzy matrices in the present form do not meet the most important requirement of matrix representation in the form of reference function without which no logical result can be expert. In this chapter, we intend to represent fuzzy matrices in which there would be the use of reference function. Our main purpose is deal especially with complement of fuzzy matrices and some of its properties when our new definition of complementation of matrices is considered. For doing these the new definition of complementation of fuzzy sets based on reference function plays a very crucial role. Further, a new definition of trace of a fuzzy matrix is introduced and also establishes some of properties.

Representation of Fuzzy Matrix:

Definition:

A representation of a fuzzy matrix A, which are defined in accordance with the existing definition would be the following,

$$A=\begin{pmatrix} 0.3 & 0.7 & 0.8 \\ 0.4 & 0.5 & 0.3 \\ 0.6 & 0.1 & 0.4 \end{pmatrix}$$

Which is a function fuzzy matrix of order 3.We would like to represent the matrix in the following manner taking into consideration of reference function.

$$A=\begin{pmatrix} (0.3,0) & (0.7,0) & (0.8,0) \\ (0.4,0) & (0.5,0) & (0.3,0) \\ (0.6,0) & (0.1,0) & (0.4,0) \end{pmatrix}$$

Then the matrix called

$$A=\begin{pmatrix} (0.3,0) & (0.7,0) & (0.8,0) \\ (0.4,0) & (0.5,0) & (0.3,0) \\ (0.6,0) & (0.1,0) & (0.4,0) \end{pmatrix}$$

Conclusion:

These models are very much used by doctors, engineers, scientists, industrialists and statisticians. This project deals with finding new definition of trace of fuzzy matrices. For doing so addition and multiplication of matrices are to be defined accordingly. For which it was required to define addition and multiplication of fuzzy matrices with the help of reference function which is different from the existing definitions. The reference function plays a key role in defining complementation of fuzzy sets and hence the presence of it is essential in dealing with complementation of fuzzy matrices.

Also, the main purpose of the project a new definition of trace of a fuzzy square matrix and some of its properties are discussed. In this project many properties of the trace of fuzzy matrices were illustrated with the help of numerical examples are studied of making the making the matter clear and simple.

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