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CONNECTED DOMSATURATION NUMBER OF A GRAPH

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Abstract:

In general, a set S of vertices of a graph G is said to be saturated by an rnatching M (where a matching M in G is a set of independent edges) if every vertex of S is incident to some edge of M. In domsaturation the concept of domination is in introduced. The domsaturation number, ds of a graph G is defined to be the least positive integer k such that every vertex of G lies in a dominating set of cardinality k. A graph G consists of a pair (V(G) , E(G)) where V(G) is a non-empty finite set whose elements are called points or vertices and E(G) is a set of unordered pairs of distinct elements of V(G). The elements of E(G) are called lines or edges of the graph G. A graph with p points and q lines is called a (p, q) graph.

Key Words: Number of Graph, Connected Graphs, etc.,

Introduction:

Within last thirty years parallel to the growth of Computer Science, Electrical and Computer Engineering and operations Research, Graph Theory is also exhibiting explosive growth. The fastest-growing area within graph theory is the study of domination in graphs. In this paper, an attempt is made to develop a concept in domination called the domsaturation number of a graph. In general, a set S of vertices of a graph G is said to be saturated by an rnatching M (where a matching M in G is a set of independent edges) if every vertex of S is incident to some edge of M. In domsaturation the concept of domination is in introduced. The domsaturation number, ds of a graph G is defined to be the least positive integer k such that every vertex of G lies in a dominating set of cardinality k.

Preliminaries:

Definition 1.1:

A graph G consists of a pair (V(G) , E(G)) where V(G) is a non-empty finite set whose elements are called points or vertices and E(G) is a set of unordered pairs of distinct elements of V(G). The elements of E(G) are called lines or edges of the graph G. A graph with p points and q lines is called a (p, q) graph.

Example 1.1:

Let V: {a,b, c, d} and E= {{a,b}, {a,c}, {a,d}} G = (V, E) is a (4,3) graph. This graph can be represented by the diagram given in Figure l.

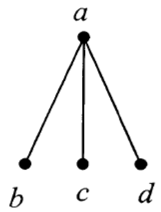

Fig 1

Definition 1.2:

If x = {u,v} ∈ E(G), the line x is said to join u and v. We write x = uv and we say that the points u and v are adjacent. We also say that the point u and the line x are incident with each other. If two distinct lines x and y are incident with a common point then they are called adjacent lines.

Example 1.2:

Let V={1,2,3,4} and E={{1,2},{1,3},{1,4},{2,3},{2,4},{3,4}}. This graph is represented by the diagram in Figure 2. In the diagram a,b, c, d, e, f represent the edges in E. Fig 2

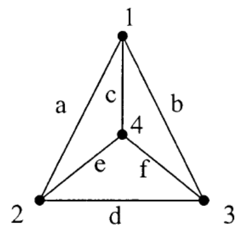

Fig 2

The line b joins the points 1 and 3 and hence the vertices 1 and 3 are adjacent.
The line b is incident with 1 as well as 3.
The lines b and c are called adjacent lines as they have 1 as a common point.

Domsaturation Number of a Graph:

Definition 2.1:

Let G = (V, E) be a graph. A subset S of V is called a dominating set if every vertex in V- S is adjacent to at least one vertex in S. The domination number $\gamma$of G is the minimum cardinality of a dominating set in G.

Example 2.1:

Consider the graph

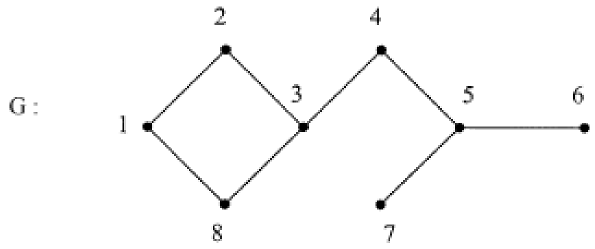

Fig 1

For this graph S_1= {1,3,5}, S_2= {3,6,7,8}, S_3= {2.,4,6,,7,8} are some of the dominating sets.
The domination number γ(G)= 3 since { 1,3,5} is a dominating set of minimum cardinality.

Definition 2.2:

The maximum order of a partition of V intodominating sets of G is called the domatic number of G and is denoted by d.

Example 2.2:

Consider the graph

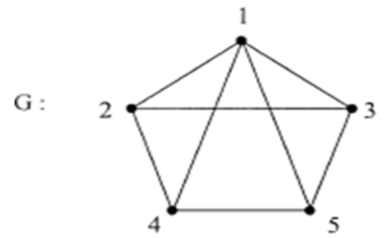

Fig 2

For this graph {{1}, {2,3}, {4,5}}, {{1}, {2,3,4,5}}, {{2,5}, {3,4},{1}} are some of the partitions of V into dominating sets. The domatic number d (G) =3 since {{1}, {2, 5}, {3, 4}} is a maximum order partition of V into dominating sets of G. In particular, d (K_n) = n

Definition 2.3:

A graph G is said to be domatically full if d = δ + 1 where δ is the minimum degree of a vertex in G.

Example 2.3:

Consider the graph

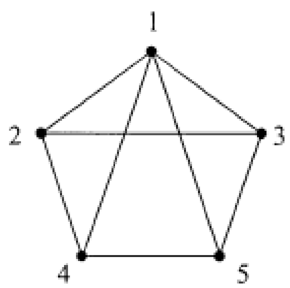

Fig 3

For this graph δ = 2 and d(G) = 3
(ie) d=2+1= δ + 1
∴ G is domatically full.

Connected Domsaturation Number of a Graph:

Definition 3.1:

The connected domsaturation number ds_c of a connected graph G = (V, E) is the least positive integer k such that every vertex of G lies in a connected dominating set of cardinality k.

Example 3.1:

Consider the graph

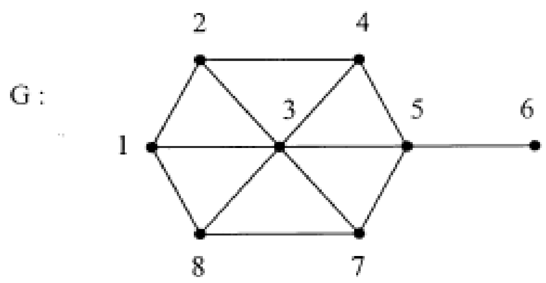

Fig 1

For this graph ds_c= 3 since { 1,3,5 }, {2,3,5 }, {4,3,5 }, {7 ,3,5 }, {8,3,5 } and {3,5,6} are connected dominating sets of cardinality 3 containing all the vertices of G and 3 is the least positive integer with this property. Let C_n and P_n denote the cycle and the path on n-vertices.

Note 3.1:

- $ds_c(k_n)=1$
- $ds_c(c_n)=n-2$
- $ds_c(p_n)=n-1$

Result 3.1:

For any graph G such that G and $\overline{G}$ are connected $\gamma_c+\overline{\gamma}_c \leq p+1$ and equality holds iff $G \cong c_5$

Result 3.2:

For any graph G such that G and $\overline{G}$ are connected, $\gamma_c+\overline{\gamma}_c = p$ iff $G \cong c_p$ $(p \geq 6)$, $P_p(p\geq 4)$,or the graph G_1,given in Fig.2.

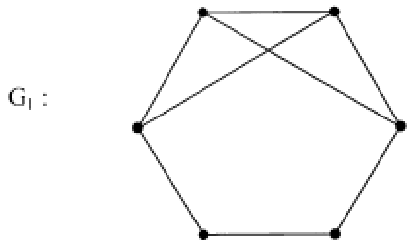

Fig 2

Result 3.3:

For any connected (p,q) graph G with maximum degree Δ, $[\frac{p}{\Delta+1}] \leq \gamma_c(G) \leq 2q-p$. The lower bound is attained iff G has a vertex of degree p - 1 and the upper bound is attained iff G is a path.

Result 3.4:

Let G be a connected graph of order $p\geq$ 4such that both G and $\overline{G}$ are connected. Then $\gamma_c+\overline{\gamma}_c \leq p\ (p-3)$. The bound is attained iff G is P_4.

Note 3.2:

If V is the vertex set of G and S is a γ_c - set of G, then for any vertex u ∈ V-S, S∪{u} is a connected dominating set and hence it follows that $ds_c= \gamma_c$ orγ_c+1.

Definition 3.2:

Let G= (V, E) be a connected graph and let v∈V. Then vertex v is said to be γ_c - totally free if v does not lie in any γ_c - set of G.

Example 3.2:

In a tree any pendant vertex is γ_c - totally free since noγ_c – set contains a pendant vertex.

Definition 3.3:

The vertex v is said to be γ_c - free if these exist γ_c - sets A and B such that v ∈ A and v ∉ B.

Example 3.3:

In a complete graph, every vertex is γ_c - free. For a complete graph K_n, {u} is a γ_c - Set for any arbitrary vertex u of K_n. Hence every vertex is γ_c - free.

Definition 3.4:

The vertex v is said to be γ_c - fixed if v lies in every γ_c - set of G.

Example 3.4:

In a tree every non-pendant veftex is γ_c - fixed. Since $< \gamma_c >$ is connected and $\gamma_c$dominates all the pendant vertices, every non-pendant veftex lie in every γ_c - set of trees.

Conclusion:

The domsaturation ds of a graph G=(V,E) is the least positive integer k such that every vertex of G lies in a dominating set of cardinality k. we obtain several result connecting ds and other graph theoretic parameters. We also characterize certain classes of graph for which ds=γ. The connected domsaturation number of ds_c of a graph G=(V,E) is the least positive integer k such that every vertex of G lies in a connected dominating set of cardinality k.

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