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EULER'S TOTIENT FUNCTION IN TERMS OF INTEGER PARTS OF RATIONAL NUMBERS

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Abstract:

We show a relation for the Euler's totient function in terms of the number of divisors function and finite sums of integer parts of rational numbers. This relation is obtained using the generalization of Menon's identity deduced by Jafari-Madadi, and it implies a recent formula of Pain.

Key Words: Divisor Function, Integer Part of a Number, Euler's Totient Function, Menon's Identity, Greatest Common Divisor, Jafari-Madadi's Formula, Pain's Expression.

1. Introduction:

From [1] we have the following expression for the greatest common divisor of $k-1$ and m:

$$(k-1, m) = 2\sum_{j=1}^{m-1}\left\lfloor\frac{j\,(k-1)}{m}\right\rfloor + (1-m)\,k + 2m - 1, \tag{1}$$

Involving the integer parts of rational numbers. On the other hand, Jafari-Madadi [2] obtained the relation:

$$\sum_{k=1,\ (k,n)=1}^{n} (k-1, m) = \varphi(n)\, d(m), \quad m\,|\,n\,, \tag{2}$$

For the Euler's totient and number of divisors functions [3], which is a generalization of the Menon's formula [4]:

$$\sum_{k=1,\ (k,n)=1}^{n} (k-1, n) = \varphi(n)\, d(n). \tag{3}$$

In Sec. 2 we use (1) and (2) to obtain $\varphi(n)$in terms of $d(m)$ and finite sums of integer parts of rational numbers.

2. Euler's Totient Function:

If $m|\,n$, then from (1) and (2):

$$\varphi(n)\, d(m) = 2\sum_{j=1}^{m-1}\sum_{k=1,\ (k,n)=1}^{n}\left\lfloor\frac{j\,(k-1)}{m}\right\rfloor + (1-m)\frac{n}{2}\,\varphi(n) + (2m-1)\,\varphi(n), \tag{4}$$

Where were applied the properties:

$$\varphi(n) = \sum_{k=1,\ (k,n)=1}^{n} 1\,, \frac{n}{2}\,\varphi(n) = \sum_{k=1,\ (k,n)=1}^{n} k\ ; \tag{5}$$

Thus (4) generates the interesting formula for the Euler's totient function:

$$\varphi(n) = \frac{4}{2\,d(m) + n\,m - 4\,m - n + 2}\sum_{j=1}^{m-1}\sum_{k=1,\ (k,n)=1}^{n}\left\lfloor\frac{j\,(k-1)}{m}\right\rfloor, \quad m\,|\,n\,, \tag{6}$$

Which implies the following Pain's expression if $m = n$:

$$\varphi(n) = \frac{4}{2\,d(n) + n^2 - 5\,n + 2}\sum_{j=1}^{n-1}\sum_{k=1,\ (k,n)=1}^{n}\left\lfloor\frac{j\,(k-1)}{n}\right\rfloor. \tag{7}$$

Remark 1:

The relation (1) can be obtained using the identity [5, 6]:

$$(k, m) + k\,m = \sum_{j=1}^{m}\left\lfloor\frac{j\,k}{m}\right\rfloor + \sum_{j=1}^{k}\left\lfloor\frac{j\,m}{k}\right\rfloor. \tag{8}$$

Remark 2:

In (2) and (3) we see connections between $\varphi(n)$ and the greatest common divisor (gcd) because the Euler's totient function is the Discrete Fourier Transform of the gcd, evaluated at 1 [7].

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